Teaching Reform of Linear Algebra Enabled by Artificial Intelligence

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Abstract: Linear algebra is a highly abstract and theoretical course, and students often encounter difficulties in conceptual understanding and in connecting theory with applications. With the rapid development of generative artificial intelligence (GenAI), new possibilities have emerged for instructional support, learning feedback, and educational assessment. Based on theories of active learning and inquiry-based instruction, this paper proposes an AI-enhanced instructional model for linear algebra organized around a closed loop of concept construction, inquiry-based learning, and formative feedback. The paper discusses the application of this model to core topics in linear algebra and analyzes the role of GenAI as a cognitive scaffold under teacher guidance. The study argues that, when appropriately regulated, GenAI can effectively support students' conceptual understanding and transferable knowledge in linear algebra.

Keywords**:** linear algebra; generative artificial intelligence; inquiry-based learning; active learning; teaching reform

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1. Introduction

Linear algebra is a foundational course for engineering, science, and data-related disciplines. Its core content includes vector spaces, linear transformations, eigenvalues, and orthogonal decompositions. However, extensive teaching practice indicates that many students focus primarily on mastering algorithms and symbolic manipulations while failing to develop a coherent understanding of underlying concepts. Previous studies suggest that this imbalance between computation and conceptual understanding is a major reason for unsatisfactory learning outcomes in linear algebra.

From a pedagogical perspective, large-scale empirical research in STEM education has demonstrated that active learning significantly improves student performance and reduces failure rates compared with traditional lecture-based instruction [1]. At the same time, generative artificial intelligence has attracted increasing attention in education due to its potential to provide instant feedback, personalized explanations, and learning support. Nevertheless, research also highlights challenges such as inaccurate outputs, student over-reliance, and risks to academic integrity (Pepin et al.[2]; Turmuzi et al.[3]). Therefore, it is of both theoretical and practical significance to explore how AI can be meaningfully integrated into linear algebra teaching within a well-regulated instructional framework.

2. Theoretical Background and Related Research

Research on linear algebra education consistently emphasizes the importance of conceptual understanding. A recent scoping review published in ZDM – Mathematics Education argues that high-quality linear algebra instruction should help students coordinate algebraic representations, geometric intuition, and application contexts, rather than focusing solely on computational skills[4].

In terms of instructional approaches, Inquiry-Oriented Linear Algebra (IOLA) emphasizes problem-driven learning tasks that guide students to construct meaning through conjecture, verification, and reflection. Design-based research has shown that such approaches effectively promote students' deep conceptual understanding and classroom engagement (Wawro et al.[5]). In addition, combining inquiry-based instruction with flipped classroom models has been found to further enhance student participation and learning depth (Fredriksen et al.[6]).

Regarding the use of generative AI in mathematics education, existing systematic studies suggest that AI can support lesson design, formative assessment, and student self-directed learning. However, the effectiveness of AI integration depends heavily on teachers' instructional design and guidance. Consequently, a hybrid model in which AI serves as a support tool under teacher supervision is widely recommended (Pepin et al.[2]; Turmuzi et al.[3]).

3. An AI-Enhanced Instructional Model for Linear Algebra

Based on the theoretical perspectives and prior studies discussed above, this paper proposes an AI-enhanced instructional model for linear algebra organized around a closed loop of concept construction, inquiry-based learning, and formative feedback. The central design principle of this model is that generative AI should function as a cognitive scaffold rather than a substitute for mathematical reasoning, with the teacher retaining primary responsibility for instructional orchestration and epistemic control.

3.1 Concept Construction with AI Support

In traditional linear algebra instruction, abstract definitions and symbolic procedures are often introduced before students develop adequate intuition, which can hinder meaningful understanding. In the proposed model, AI is used to support the initial construction of concepts by generating multi-representational explanations. These explanations may include algebraic formulations, geometric interpretations, verbal descriptions, and brief application-oriented contexts.

To prevent students from passively accepting AI-generated content, teachers design structured learning tasks that require students to actively engage with these materials. For example, students may be asked to paraphrase AI explanations in their own words, identify underlying assumptions, and construct simple examples or counterexamples. This process aligns with research emphasizing the importance of student-generated representations and explanations in conceptual learning.

Importantly, AI outputs are treated as tentative resources rather than authoritative knowledge. Students are required to verify claims using textbooks, lecture notes, or classroom discussions, which helps cultivate critical evaluation skills and reduces the risk of uncritical reliance on AI-generated responses.

3.2 Inquiry-Based Learning in the Classroom

Inquiry-based learning constitutes the core of classroom activity in the proposed model. Building on the Inquiry-Oriented Linear Algebra (IOLA) tradition, lessons are structured around carefully designed tasks that prompt students to explore mathematical ideas through conjecture, experimentation, and justification. Typical tasks focus on central concepts such as the meaning of linear transformations, the interpretation of eigenvalues as invariant directions, or the role of orthogonality in projection and approximation. Within this phase, AI plays a deliberately constrained role. For instance, AI may ask students to clarify assumptions, consider special cases, or explain why a particular argument fails. Such uses of AI support productive struggle and mathematical discourse, both of which are essential for deep learning. Teachers facilitate discussion, monitor group interactions, and intervene when necessary to maintain mathematical rigor. This division of roles ensures that AI enhances, rather than undermines, the social and cognitive dimensions of inquiry-based learning.

3.3 Formative Feedback and Learning Analytics

The final component of the instructional loop is formative feedback. In large-enrollment linear algebra courses, providing timely and detailed feedback is often challenging. AI can assist by generating rubric-based comments on students' written work, focusing on conceptual accuracy, logical coherence, use of notation, and clarity of explanation. In addition to individual feedback, AI-supported analysis of student responses can help identify common misconceptions and error patterns, such as confusion between eigenvalues and eigenvectors or misinterpretation of matrix rank. Teachers can use this information to adjust subsequent instruction, design targeted review activities, or revisit challenging concepts.

Crucially, summative assessment remains the responsibility of the instructor. AI-generated feedback is used formatively, supporting learning rather than determining grades. This approach aligns with recommendations from recent studies on the responsible integration of generative AI in mathematics education (Pepin et al.[2]; Turmuzi et al.[3]).

4. Instructional Applications and Evaluation Considerations

4.1 Applications to Core Topics in Linear Algebra

The proposed AI-enhanced instructional model is adaptable to multiple core topics in a standard linear algebra curriculum. In teaching matrices and linear transformations, students are encouraged to interpret matrices as functions acting on vectors rather than as mere arrays of numbers. Tasks may involve reconstructing matrices from the images of basis vectors or comparing the effects of different transformations on geometric objects. AI supports this process by prompting students to articulate connections between algebraic operations and geometric meaning. In the unit on eigenvalues and eigenvectors, instruction emphasizes their conceptual interpretation as invariant directions or scaling behaviors under linear transformations. Students explore conditions for diagonalizability through examples and counterexamples, guided by inquiry-based tasks. AI can assist by generating contrasting cases or by highlighting missing logical links in students' explanations, without revealing complete solutions.

4.2 Evaluation Strategies

Evaluation within the proposed framework prioritizes conceptual understanding, reasoning quality, and mathematical communication. Rather than relying solely on traditional problem sets or examinations, a combination of assessment methods is recommended. Conceptual assessments may include short-answer or explanation-based questions administered before and after instructional units to gauge conceptual growth. Iterative assignments, in which students revise their work based on AI and instructor feedback, provide insight into learning processes and the development of understanding over time. Classroom observations and participation in group discussions offer additional qualitative evidence of student engagement and reasoning. Importantly, evaluation design also addresses concerns related to academic integrity and over-reliance on AI. Oral explanations, in-class reasoning tasks, and justification-based questions can be used to verify students' independent understanding. Such strategies are consistent with prior research on AI-supported linear algebra instruction and help ensure the validity of assessment results.

5. Conclusion

This paper has examined the integration of generative artificial intelligence into the teaching of linear algebra through an instructional model structured around concept construction, inquiry-based learning, and formative feedback. Unlike purely procedural mathematics courses, linear algebra is characterized by its high level of abstraction, its reliance on structural reasoning, and the need to coordinate multiple representations, including symbolic, geometric, and functional perspectives. These features make linear algebra particularly challenging for students, but also especially suitable for carefully designed AI-supported instruction.

From the perspective of linear algebra content, the proposed model directly targets long-standing learning difficulties. AI-supported concept construction helps students move beyond viewing matrices as computational objects toward understanding them as representations of linear transformations. Inquiry-based activities, supported by constrained AI questioning, encourage students to reason about invariant subspaces, eigenvalues as scaling factors, and orthogonality as a structural principle underlying projection and approximation. In this way, AI is used not to accelerate routine computation, but to support the development of structural insight and mathematical meaning, which are central to linear algebra.

Furthermore, the emphasis on formative feedback is well aligned with the nature of linear algebra learning. Many conceptual obstacles in this course—such as confusion between vectors and scalars, misinterpretation of rank and dimension, or fragmented understanding of diagonalization—are not easily resolved through summative assessment alone. AI-assisted feedback, when guided by carefully designed rubrics, can help students articulate their reasoning, identify conceptual gaps, and iteratively refine their understanding of key ideas. This approach supports the gradual construction of a coherent conceptual framework, which is essential for success in linear algebra and its applications in advanced mathematics, engineering, and data science.

Several limitations should be acknowledged. The instructional model presented in this paper is conceptual and requires empirical validation in authentic linear algebra classrooms. Future research may involve design-based studies examining how different AI scaffolding strategies influence students' understanding of core linear algebra concepts, such as linear independence, diagonalization, and orthogonal decomposition. Investigating how students' mathematical thinking evolves when learning linear algebra with AI tools also represents an important direction for further inquiry.

In conclusion, generative artificial intelligence holds promise for supporting teaching reform in linear algebra precisely because of the course's conceptual density and structural coherence. When embedded within an inquiry-oriented and concept-focused pedagogical framework, AI can help students engage more deeply with the fundamental ideas of linear algebra, while preserving the essential role of mathematical reasoning and human instruction.

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