# A Study on the Design of Ideological and Political Teaching Cases in Linear Algebra

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**Abstract:** Linear Algebra is a foundational course for science, engineering, economics, computer science, and data-related majors. However, in many classrooms it is still taught mainly through definitions, theorems, and mechanical calculation, which may weaken students' understanding of its social value and humanistic significance. In the context of China's curriculum-based ideological and political education, this paper explores how ideological and political elements can be naturally integrated into Linear Algebra through case-based teaching design. Based on constructive alignment, situated learning, and active learning theory, the study proposes a teaching case design framework consisting of "mathematical knowledge, application scenario, value implication, learning activity, and assessment evidence." Three sample cases are discussed: matrix representation and national infrastructure planning, linear systems and scientific decision-making, and eigenvalues/eigenvectors and technological innovation. The paper argues that ideological and political education in Linear Algebra should not be added mechanically, but should arise from the internal logic of mathematical knowledge, the history of scientific development, and real social applications. Such design can improve students' mathematical competence, social responsibility, scientific spirit, and confidence in applying mathematics to national and human development.

***Keywords*:** Linear Algebra; curriculum ideology and politics; teaching case design; mathematical education; value education

## 1. Introduction

Curriculum-based ideological and political education, often called "curriculum ideology and politics" in Chinese higher education, emphasizes that all university courses should contribute to moral cultivation while maintaining their disciplinary characteristics. The Ministry of Education of the People's Republic of China states that higher education courses should integrate knowledge transmission, ability cultivation, and value guidance. For mathematics courses, this requirement does not mean replacing mathematical rigor with political preaching. Rather, it requires teachers to reveal the cultural, scientific, ethical, and social meanings embedded in mathematical concepts and applications.

Linear Algebra is particularly suitable for this purpose. Matrices, vectors, linear transformations, systems of equations, orthogonality, eigenvalues, and vector spaces are not only abstract mathematical objects but also essential tools in engineering design, artificial intelligence, network analysis, economics, image processing, and national strategic technologies. As Strang shows, Linear Algebra has become a language for modern science and computation [1]. Therefore, teaching Linear Algebra can help students understand both the beauty of mathematical abstraction and the responsibility of using mathematical methods to solve real problems.

This paper focuses on one question: how can ideological and political teaching cases be designed in a Linear Algebra course without damaging mathematical depth? The aim is to provide a practical framework and several examples for classroom implementation.

## 2. Theoretical Basis

First, the design follows the principle of constructive alignment. Biggs and Tang argue that learning objectives, teaching activities, and assessment tasks should be aligned [2]. In Linear Algebra, ideological and political elements should therefore correspond to specific learning outcomes, rather than appearing as unrelated classroom slogans. In addition, the classification of learning objectives proposed by Anderson and Krathwohl provides useful support for designing outcomes involving knowledge and ability [3].

Second, situated learning theory emphasizes that knowledge is learned more effectively in meaningful contexts. Lave and Wenger point out that learning occurs through participation in social practices [4]. When students encounter matrices in transportation planning, data security, public health, or digital economy contexts, they can better understand why abstract mathematical tools matter. Kolb's theory of experiential learning also suggests that students develop understanding through a cycle of experience, reflection, conceptualization, and practice [5].

Third, active learning supports student-centered classroom practice. Prince reviewed evidence showing that active learning improves student engagement and learning outcomes in engineering education [6]. In Linear Algebra, case discussion, modeling tasks, group presentations, and reflective writing can connect mathematical reasoning with ethical and social reflection.

Finally, the design is consistent with the broader goal of liberal and professional education. Freire argued that education should develop critical consciousness rather than passive acceptance [7]. For mathematics education, this means guiding students to ask not only "How do we calculate?" but also "Why is this model useful?" "What assumptions does it contain?" and "How should mathematical technology serve society?"

# 3. A Framework for Teaching Case Design

A Linear Algebra ideological and political teaching case can be designed through five connected components. The first component is mathematical knowledge. Each case must begin from core course content, such as matrix multiplication, rank, linear independence, eigenvalues, orthogonal projection, or diagonalization. This ensures that value education is grounded in disciplinary learning.The second component is the application scenario. The teacher selects a realistic or historically meaningful problem, such as transportation networks, BeiDou navigation, image compression, intelligent manufacturing, epidemic data analysis, or ecological monitoring. The scenario should be understandable to students and mathematically connected to the lesson.The third component is value implication. The case should guide students toward scientific spirit, national responsibility, engineering ethics, innovation awareness, cooperation, or cultural confidence. The value element should be implicit in the problem situation and made explicit through questions and reflection.The fourth component is learning activity. Students may calculate, model, compare methods, interpret results, debate model limitations, or write short reflections. The activity should require both mathematical work and value-based thinking. The fifth component is assessment evidence. Teachers may assess students' mathematical accuracy, modeling process, explanation, teamwork, and reflection. This prevents ideological and political education from becoming vague or unmeasurable.

# 4. Sample Teaching Cases

Case 1: Matrix Representation and National Infrastructure Planning

This case can be used when teaching matrices and matrix multiplication. The teacher introduces a simplified transportation network among several cities. A matrix represents direct railway or logistics connections, while powers of the matrix indicate possible transfer routes. Students calculate matrix products and interpret the meaning of nonzero entries. The mathematical objective is to understand matrix multiplication as composition of relations, not merely as a formula. The application objective is to recognize how mathematical structures support planning in railway systems, logistics networks, and urban development. The ideological and political objective is to help students understand that national infrastructure construction requires systematic thinking, precision, cooperation, and long-term planning. Classroom questions may include: What does the entry in the second power of the adjacency matrix mean? How can matrix methods help planners identify important transportation nodes? What risks might appear if decision-makers only consider efficiency and ignore fairness or regional balance? Through these questions, students connect linear algebra with coordinated regional development and public service.

Case 2: Linear Systems and Scientific Decision-Making

This case is suitable for teaching systems of linear equations, Gaussian elimination, rank, and solution structures. The teacher presents a simplified resource allocation problem, such as distributing emergency materials to several regions under limited supply and transportation constraints. Students formulate the constraints as a linear system and analyze whether the system has no solution, a unique solution, or infinitely many solutions. The mathematical focus is rank, consistency, and the relationship between equations and unknowns. The value focus is scientific decision-making. Students learn that responsible public decisions should be based on evidence, constraints, transparency, and rational analysis. At the same time, the teacher can emphasize that mathematical models are simplifications of reality. A feasible solution in algebra may still need ethical review, local knowledge, and human care. This case also cultivates humility in using mathematics. Students should understand that mathematics is powerful, but it does not automatically replace social judgment. Such awareness is important in the era of big data and artificial intelligence, where algorithms increasingly influence public decisions.

Case 3: Eigenvalues, Eigenvectors, and Technological Innovation

Eigenvalues and eigenvectors are often difficult for students because of their abstract nature. A case based on search engines, recommendation systems, image recognition, or vibration analysis can help students see their importance. For example, when introducing PageRank, the teacher can explain that a web network can be represented by a matrix, and the importance of a webpage can be modeled through a special eigenvector. Brin and Page demonstrated the significance of this idea in early web search technology. The mathematical objective is to understand eigenvectors as directions preserved by linear transformations. The application objective is to connect eigenvalue methods with digital technology. The ideological and political objective is to cultivate innovation awareness and technological self-reliance. Students may discuss how basic mathematics supports major technological breakthroughs, why original innovation requires long-term accumulation, and how young people can contribute to scientific and technological development. The teacher should also guide ethical reflection. Recommendation algorithms and ranking systems may affect information access and public opinion. Therefore, mathematical innovation should be combined with responsibility, fairness, and social awareness.

## 5. Implementation Strategies

First, teachers should avoid superficial insertion of ideological content. A good case should follow the natural path from concept to application and then to value reflection. For example, when teaching orthogonality, teachers can discuss signal separation or data compression; when teaching vector spaces, they can discuss abstraction and unity in scientific thinking.

Second, teachers should use short but frequent value-guiding moments. In a 45- or 90-minute class, ideological and political discussion does not need to occupy much time. A carefully designed question, a historical note, or a two-minute reflection can be effective if it is closely related to the mathematical content.

Third, teaching should combine individual calculation with group discussion. Students first complete the mathematical task, then discuss the meaning and limitations of the model. This sequence prevents the class from becoming empty discussion without mathematical foundation.

Fourth, assessment should include both knowledge and reflection. A rubric may include four dimensions: correctness of mathematical method, clarity of modeling, awareness of assumptions and limitations, and quality of value reflection. Reflection tasks should be concise, such as a 150-word response on how a mathematical model can serve society responsibly.

## 6. Conclusion

This paper proposes a teaching case design approach for integrating ideological and political education into Linear Algebra. The key principle is disciplinary integration: value education should grow from mathematical concepts, historical development, practical applications, and reflective activities. Through cases on matrix representation, linear systems, and eigenvalue methods, students can learn Linear Algebra while developing scientific spirit, social responsibility, innovation awareness, and ethical judgment. In this sense, Linear Algebra teaching can contribute not only to professional competence but also to the broader educational mission of cultivating capable and responsible citizens. The integration of ideological and political education into Linear Algebra has several advantages. It can increase students' motivation by showing the relevance of abstract concepts. It can also improve students' ability to transfer knowledge from textbook exercises to real problems. More importantly, it helps students form a responsible view of mathematics: mathematics is not only a tool for calculation, but also a way of understanding, constructing, and improving the world. However, challenges remain. Some teachers may lack interdisciplinary cases; some students may regard value discussion as unrelated to mathematics; and assessment of ideological outcomes can be difficult. To address these problems, departments should build shared teaching case libraries, organize teacher training, and encourage collaboration between mathematics teachers and teachers from engineering, computer science, economics, and humanities. Case design should also be continuously improved through classroom feedback and student learning evidence.

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