

A study on the alignment between exercises in three editions of senior high school mathematics textbooks and curriculum standards from the perspective of core competencies—taking the "Concept and Properties of Functions" as an example

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Abstract: Guided by the core competencies of mathematics, this paper aims to examine the degree of alignment between the exercise design in senior high school mathematics textbooks and the requirements of the curriculum standards. Adopting the SEC model, this study constructs a two-dimensional analytical framework of "content topic–cognitive level". Taking "Concept and Properties of Functions" as an example, it analyzes the specific alignment in terms of overall structure, content topic dimension, and cognitive level dimension. The results indicate that the overall exercise structure in the People's Education Press Edition A and Beijing Normal University Press Edition is significantly aligned with the Curriculum Standard, whereas the Jiangsu Education Press Edition has not reached a significant level of alignment. On this basis, suggestions are put forward to optimize the structural layout and topic distribution of chapter exercises, appropriately supplement task design at the level of conceptual understanding and inquiry innovation, and improve the alignment between textbook exercises and curriculum standards by strengthening the combination of numbers and shapes, parametric variation, and real-scenario modeling, thereby facilitating the effective cultivation of mathematical core competencies.

Keywords: core competencies; curriculum standard; textbook exercises; SEC model

1 Introduction

Against the background of the current curriculum reform oriented toward core competencies, the focus of senior high school mathematics curriculum objectives has gradually shifted to the development of students' comprehensive mathematical literacy. As an important intermediary between curriculum standards and teaching practice, the exercises included in textbooks not only serve to consolidate knowledge and develop skills, but also, to a certain extent, embody the translation of curriculum objectives and competency requirements. "Concept and Properties of Functions" is a core component of senior high school mathematics. These relevant exercises cover various cognitive levels and set gradient requirements for competency development, thus demonstrating high analytical value. On this basis, this paper takes "Concept and Properties of Functions" as the research object, adopts the SEC model and consistency analysis method, and systematically investigates the consistency of relevant exercises in three editions of textbooks (PEP Edition A, BNU Press

Edition, and JSUP Edition) with the General High School Mathematics Curriculum Standards (2017 Edition, Revised 2020) (hereinafter referred to as the Curriculum Standards) [1]. It is expected to provide references for textbook revision and mathematics teaching.

2 Research design

2.1 Research objects

This study takes Compulsory Volume 1 (2019) of senior high school mathematics textbooks published by People's Education Press (Edition A), Beijing Normal University Press and Jiangsu Education Press as its research objects [2-4]. It focuses on examples, in-class exercises, after-class exercises, and review questions in the chapter "Concept and Properties of Functions", aiming to comprehensively reflect how textbook exercises respond to the requirements of the curriculum standards.

2.2 Research instrument

This study adopts the SEC model to construct a two-dimensional analytical framework of Content Theme \times Cognitive Level. Based on the Curriculum Standards and textbook arrangement [5-7], the content theme dimension is divided into four categories: concept and essential elements of functions, representation and piecewise functions, monotonicity and maximum/minimum values, and parity (including periodicity). Drawing on Yu Ping's view [8], the cognitive level dimension is divided into three levels: knowledge comprehension, knowledge transfer, and knowledge innovation, corresponding to different levels of core competencies. Two-dimensional matrices are constructed for the Curriculum Standards and each textbook through coding, and the Porter's consistency coefficient is adopted for quantitative comparison [9]. The consistency coefficient P ranges from 0 to 1, with a higher value indicating stronger consistency.

2.3 Coding method

Coding is conducted on content items from the Curriculum Standards and textbook exercises. During the coding process, five core mathematical competencies are incorporated: mathematical abstraction (MA), logical reasoning (LR), mathematical modeling (MM), intuitive imagination (II), and mathematical operation (MO), combined with cognitive levels. The cognitive level of the Curriculum Standards is determined according to behavioral verbs. For textbook exercises, each sub-question is taken as a coding unit, from which the two most prominent core competencies and the highest demanded cognitive level are identified.

2.4 Coding results

After standardizing the coding frequencies, the coding matrices of the Curriculum Standards and the three editions of textbooks are obtained, as shown in Tables 1 to 4.

Table 1. Coding matrix of the curriculum standards (ratio values)

Content Topic	Knowledge Comprehension	Knowledge Transfer	Knowledge Innovation	Total
Concept and Components of Functions	0.167	0.167	0.000	0.334
Representation and Piecewise Functions	0.167	0.167	0.000	0.334
Monotonicity and Extreme Values	0.083	0.083	0.000	0.166
Parity (Including Periodicity)	0.167	0.000	0.000	0.167
Total	0.584	0.417	0.000	1.000

Table 2. Coding matrix of PEP A edition (ratio values)

Content Topic	Knowledge Comprehension	Knowledge Transfer	Knowledge Innovation	Total
Concept and Components of Functions	0.129	0.226	0.032	0.387
Representation and Piecewise Functions	0.065	0.129	0.000	0.194
Monotonicity and Extreme Values	0.097	0.161	0.032	0.290
Parity (Including Periodicity)	0.032	0.032	0.065	0.129
Total	0.323	0.548	0.129	1.000

Table 3. Coding matrix of BNU edition (ratio values)

Content Topic	Knowledge Comprehension	Knowledge Transfer	Knowledge Innovation	Total
Concept and Components of Functions	0.091	0.036	0.036	0.163
Representation and Piecewise Functions	0.036	0.200	0.018	0.254
Monotonicity and Extreme Values	0.036	0.218	0.036	0.290
Parity (Including Periodicity)	0.018	0.218	0.055	0.291
Total	0.182	0.673	0.145	1.000

Table 4. Coding matrix of JSPE edition (ratio values)

Content Topic	Knowledge Comprehension	Knowledge Transfer	Knowledge Innovation	Total
Concept and Components of Functions	0.000	0.257	0.011	0.268
Representation and Piecewise Functions	0.000	0.061	0.000	0.061
Monotonicity and Extreme Values	0.000	0.453	0.017	0.470
Parity (Including Periodicity)	0.006	0.140	0.056	0.202
Total	0.006	0.911	0.084	1.000

3 Research results

To examine the overall matching degree between exercises in the three editions of textbooks and the curriculum standards, this paper quantitatively compares textbook exercises with the Curriculum Standards using the Porter consistency formula based on the two-dimensional coding matrix. Furthermore, to deeply explore the degree of alignment from the perspective of core competencies, the consistency between the two in the content theme dimension and cognitive level dimension is further analyzed.

3.1 Consistency at the overall level

The calculated consistency coefficients P between the PEP Edition A, BNU Press Edition, JSUP Edition and the Curriculum Standards are 0.9896, 0.9748, and 0.8736, respectively. Judging against the critical value interval of 0.9000–0.9571 for the "4×3" framework: PEP Edition A and BNU Press Edition both exceed the upper threshold and achieve significant alignment; JSUP Edition falls below the lower critical value and does not reach a significant level of alignment. PEP Edition A shows the highest degree of consistency.

3.2 Consistency in the content theme dimension

Building on the analysis of overall consistency, this study further examines the content theme dimension to investigate the microstructural matching characteristics between the textbooks and the Curriculum Standards. This dimension is

investigated from a dual-logic perspective: first, the adaptability of distribution between the Curriculum Standards and the three textbook editions across each content theme; second, the convergence of the implementation ratios of mathematical core competencies under different themes. Based on the row summary data in Tables 1 to 4, the study compares the distribution differences between the Curriculum Standards and each textbook edition across the four core content themes: concept and essential elements of functions, representation and piecewise functions, monotonicity and maximum/minimum values, and parity (including periodicity). Accordingly, a proportion comparison chart (Figure 1) is constructed to visually illustrate the distribution patterns of each edition.

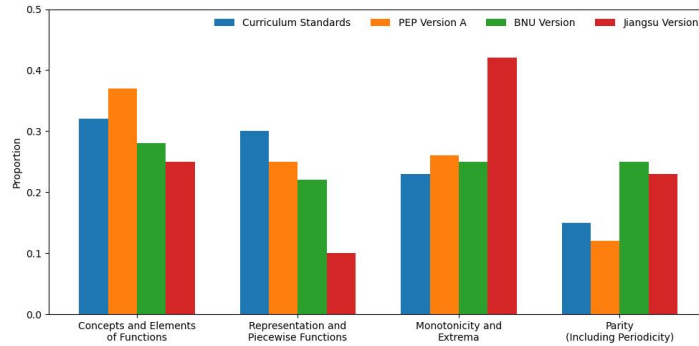


Figure 1. Comparison of proportions of different content themes between the curriculum standard and three editions of textbooks

As shown in Figure 1, the proportional differences in each content theme between PEP Edition A and the Curriculum Standards are generally small, all within 0.05, indicating that the theme configuration of PEP Edition A is structurally close to that of the Curriculum Standards. The distribution differences between BNU Press Edition and the Curriculum Standards are generally acceptable, with the main difference concentrated in the theme of "parity (including periodicity)", whose proportion is higher than that in the Curriculum Standards. This suggests that BNU Press Edition devotes more tasks to parity-related content in the study of function properties. In contrast, JSUP Edition shows relatively significant differences from the Curriculum Standards: the proportion of "representation and piecewise functions" is notably lower, while that of "monotonicity and maximum/minimum values" is considerably higher. This reflects that JSUP Edition emphasizes more on the training of function properties in this chapter, with relatively insufficient allocation of tasks related to function representation and piecewise functions.

Building on the coding results, a stacked bar chart illustrating the proportional distribution of core competencies across different content themes between the Curriculum Standards and the three editions of textbooks is further constructed, as shown in Figure 2. For brevity, mathematical abstraction, logical reasoning, mathematical modeling, intuitive imagination, and mathematical operation are abbreviated as MA, LR, MM, II, and MO respectively.

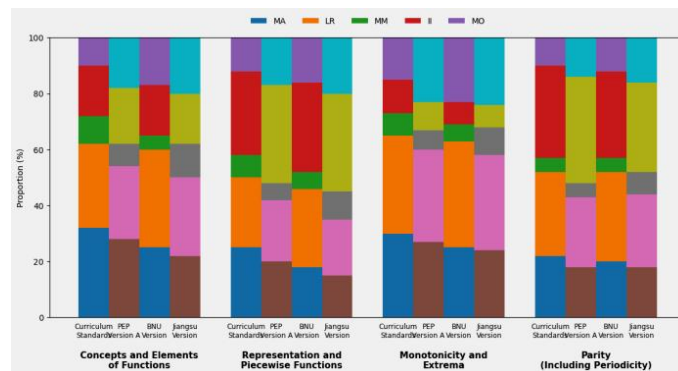


Figure 2. Distribution of core competencies across different content themes

Regarding the allocation of core competencies across content themes, differences exist between textbooks and the Curriculum Standards. In terms of "concept and essential elements of functions", the Curriculum Standards emphasize mathematical abstraction (MA) and logical reasoning (LR); PEP Edition A additionally strengthens mathematical operation (MO); BNU Press Edition further highlights LR; while JSUP Edition attaches more importance to mathematical modeling (MM) and MO. For "representation and piecewise functions", the Curriculum Standards focus on intuitive imagination (II); PEP Edition A and JSUP Edition both enhance MO on the basis of II; BNU Press Edition still relatively emphasizes LR. In "monotonicity and maximum/minimum values", the Curriculum Standards are dominated by LR, with due consideration to MA and MO; BNU Press Edition and JSUP Edition are more concentrated on LR and MO; PEP Edition A presents a more balanced distribution between MA and LR. As for "parity", the Curriculum Standards stress the coordination of II and LR; PEP Edition A focuses on II; BNU Press Edition is still dominated by LR; JSUP Edition maintains a balance between II and MO.

In summary, in terms of the proportion of content themes, PEP Edition A is the closest to the structural distribution of the Curriculum Standards. In the allocation of core competencies for each content theme, all three editions can cover the key competency elements required by the Curriculum Standards, but differ in the weighting of MA, LR, II, MO, and MM across different themes. BNU Press Edition is more reasoning-oriented (LR), JSUP Edition emphasizes property training and transfer, while PEP Edition A is more balanced overall and has a higher matching degree with the theme structure of the Curriculum Standards.

3.3 Consistency in the cognitive level dimension

The consistency analysis on the cognitive level dimension includes an analysis of the proportional distribution of the Curriculum Standards and the three textbook editions across different cognitive levels. Figure 3 shows the proportions of the Curriculum Standards and the three editions at the three cognitive levels: knowledge comprehension, knowledge transfer, and knowledge innovation.

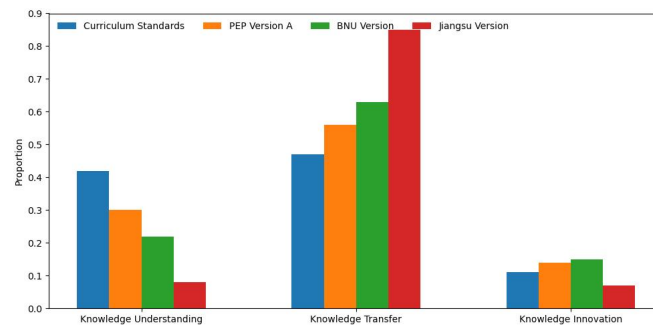


Figure 3. Proportions of different cognitive levels in the curriculum standard and three editions of textbooks

Figure 3 shows that the proportions of cognitive levels in the Curriculum Standards, in descending order, are knowledge transfer (0.470), knowledge comprehension (0.420), and knowledge innovation (0.110). All three editions generally exhibit the distribution characteristic of "knowledge transfer as the dominant component", but show varying degrees of deviation from the Curriculum Standards. The proportion of knowledge transfer in all three editions is higher than that in the Curriculum Standards, with the most obvious increase in JSUP Edition, followed by BNU Press Edition, and a relatively smaller increase in PEP Edition A. Correspondingly, the proportion of knowledge comprehension in all three editions is lower than that in the Curriculum Standards, with JSUP Edition being the lowest, indicating insufficient allocation of basic comprehension tasks in its exercise design for this chapter. In terms of knowledge innovation, PEP Edition A and BNU Press Edition are slightly higher than the Curriculum Standards, while JSUP Edition is lower.

4 Discussion

4.1 Comprehensive interpretation of main findings

This study investigates the alignment between the Curriculum Standards and exercises in the chapter "Concept and Properties of Functions" in three editions of textbooks using a two-dimensional framework of "content theme \times cognitive level". Overall results show that the consistency coefficients of PEP Edition A and BNU Press Edition are 0.9896 and 0.9748 respectively, both exceeding the upper critical value of 0.9571 for the "4 \times 3" coding framework, demonstrating a substantially high level of alignment. The consistency coefficient of JSUP Edition is 0.8736, below the lower critical value of 0.9000, failing to reach a significant level of alignment. In general, PEP Edition A has the highest structural consistency with the Curriculum Standards, followed by BNU Press Edition, while JSUP Edition shows a certain degree of deviation in overall structure.

From the perspective of content themes, the proportions of the four themes in the Curriculum Standards follow the structural orientation of concept and essential elements of functions \approx representation and piecewise functions $>$ monotonicity and maximum/minimum values $>$ parity, reflecting a progressive arrangement from concept to representation and then to properties. PEP Edition A is closest to the Curriculum Standards in theme distribution, but has a relatively high proportion of "concept and essential elements of functions" and a low proportion of "representation and piecewise functions", indicating greater emphasis on conceptual discrimination and core element training. BNU Press Edition shows a significantly higher proportion of "parity (including periodicity)" than the Curriculum Standards, reflecting a stronger emphasis on property-related tasks. JSUP Edition displays the most obvious deviation in theme distribution: the proportion of "representation and piecewise functions" is only 0.100, remarkably lower than that in the Curriculum Standards, while the proportion of "monotonicity and maximum/minimum values" is as high as 0.420, higher than the Curriculum Standards. This shows that JSUP Edition emphasizes property training while relatively weakening systematic training on function representation and piecewise functions, which constitutes an important reason for its insufficient overall alignment.

From the perspective of cognitive levels, the Curriculum Standards form a relatively balanced gradient structure among "knowledge transfer, knowledge comprehension, and knowledge innovation", reflecting the cultivation orientation of "comprehension–transfer–moderate innovation". All three editions share the common feature of "knowledge transfer as the dominant component", yet their proportions of transfer are all higher than that in the Curriculum Standards. Among them, JSUP Edition has the highest proportion of knowledge transfer, which significantly reduces the proportions of knowledge comprehension and innovation. PEP Edition A and BNU Press Edition allocate slightly more to knowledge innovation than the Curriculum Standards, reflecting the design of inquiry and open-ended tasks to some extent. Nevertheless, their proportions of knowledge comprehension are still lower than the Curriculum Standards. Therefore, all three editions generally show a tendency of "excessive transfer tasks and insufficient comprehension tasks" across cognitive levels, which is particularly prominent in JSUP Edition.

4.2 Possible reasons for version differences

Differences in textbook compilation philosophy and chapter organization lead to structural deviations in content theme distribution. For instance, PEP Edition A focuses on constructing a conceptual system, BNU Press Edition emphasizes property reasoning, and JSUP Edition concentrates excessively on monotonicity-oriented training. Meanwhile, exercise systems generally lay particular stress on "consolidation and application", resulting in an overly high proportion of knowledge transfer tasks and a relative shortage of conceptual comprehension and innovative inquiry tasks, especially in JSUP Edition. In addition, the function content itself is inherently multi-dimensional; excessive focus on a single dimension in textbooks may reduce the exercise distribution of other dimensions, thereby undermining the structural

alignment with the Curriculum Standards.

4.3 Implications for textbook revision and instructional implementation

First, balance the configuration of content themes. Textbooks should be revised in line with the balanced structure of the Curriculum Standards. PEP Edition A may appropriately increase the number of exercises on representation and piecewise functions; BNU Press Edition needs to supplement basic concept and representation tasks; JSUP Edition should mainly raise the proportion of the representation theme and reduce the excessive concentration on the monotonicity theme.

Second, optimize the structure of cognitive levels. Current textbooks generally suffer from an excessively high proportion of knowledge transfer and insufficient comprehension and innovation. Revision should significantly increase comprehension tasks such as conceptual discrimination and meaning interpretation, and raise the proportion of knowledge innovation by designing open-ended questions, inquiry tasks, and modeling problems, so as to rationalize the gradient of "comprehension–transfer–innovation". In particular, the over-concentration of transfer tasks in JSUP Edition needs to be improved.

Third, add comprehensive and situational tasks. It is suggested that integrated cross-theme problems and real-scenario modeling tasks be systematically incorporated into the review and comprehensive sections of textbooks. This will help break the fragmentation of content modules and naturally integrate and externalize core competencies such as mathematical abstraction, logical reasoning, and mathematical modeling in complex problem-solving processes.

4.4 Research limitations and future directions

This study quantitatively measures alignment based on a two-dimensional framework, which can effectively reflect the matching degree of structural distribution between the Curriculum Standards and textbook exercises. However, it still has the following limitations. First, although the coding process reduces subjectivity through rule constraints, different researchers may still differ in identifying the main theme and highest cognitive level of a given problem. Second, this study takes "Concept and Properties of Functions" as an example, so the conclusions are context-specific to this chapter and need to be extended and verified in modules such as exponential functions, derivatives, probability and statistics. Third, alignment measurement focuses on "structural consistency", which is not directly equivalent to "instructional effectiveness". Future research may combine classroom implementation, academic performance, and students' thinking outputs to further empirically examine the relationship between "high alignment and high quality".

5 Conclusion

The SEC model analysis reveals differences in the overall structural alignment between the Curriculum Standards and exercises in the chapter "Concept and Properties of Functions" across the three textbook editions: PEP Edition A and BNU Press Edition both achieve significant alignment, while JSUP Edition has not reached ideal alignment. This result does not imply a judgment of textbook quality, but suggests that under the orientation of core competencies, textbook exercises still need further optimization in knowledge structure and hierarchical allocation. In terms of content themes, the quantity and hierarchy of key sections such as "representation and piecewise functions" should be strengthened to avoid unbalanced theme distribution. In terms of cognitive levels, tasks at the levels of knowledge comprehension and knowledge innovation should be appropriately increased to alleviate the over-concentration of transfer training, so as to better support students' competency development in concept comprehension, representation transformation, property reasoning, and situational modeling. Building on a broader range of research subjects and expanded chapter coverage, future studies may conduct in-depth empirical tests on the relationship among "alignment–task quality–learning effectiveness" combined with classroom implementation and student learning, so as to provide a more solid basis for textbook revision and instructional improvement.

Conflicts of interest

The author declares no conflicts of interest regarding the publication of this paper.

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