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Virtual rotodynamic pumps method

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Abstract: The method of virtual rotodynamic pumps is presented, a simple and practical procedure for the solution of pump problems connected in parallel, when the installation conditions and/or characteristics of the coupled pumps are not the same. Virtual pumps are imaginary machines that lack secondary suction and impulse pipes. These pumps are formed by the real pump supplied by the manufacturer and all accessories and pipes to the point of binding with the main drive pipe. These teams are always characterized by working against the same total burden when they operate in parallel.

Key words: pumps; rotodynamic; virtual; method

1 Introduction

In different applications of rotodynamic pumps, it is necessary to analyze different operating options, to meet the needs that are presented. One of these options is to combine pumps of equals or different characteristics in parallel. This operation scheme is mainly done to fraction a total flow in partial flows, when working in a variable spending and constant demand or to increase the flow in a pumping system (Pardo and Ruiz 1980, Pérez 2012).

It is known that, in general terms, if the pumps placed in parallel are the same, the capacity and power necessary for equal loads will double, triple, etc.; Depending on two, three or more equal pumps placed in parallel, and the efficiency of the set will be equal to that of the original pump. This working condition will only be fulfilled, when secondary pipe systems (suction and impulse of each pump), and the installed pumps are equal (Pardo and Ruiz 1980, Karassik et al. 1986). For any other situation, the statement raised in the previous paragraph will not be fulfilled and adjustment for the calculation of the real operating point (Martínez and Riaño 2010) must be made.

As a fundamental characteristic to highlight in a parallel pump coupling, it is that the total flow delivered by the combination is always less than the number of pumps by the flow that supplies a single pump (Cherkasski 1986). The existing relationship between the flow that supplies a pump and the one that supplies the combination will depend on the characteristics of each of the coupled pumps, the set in parallel and the characteristic curve of the pipe system (Martínez and Riaño 2010).

Rotodynamic pumps operating in parallel are influenced by mutually: the flow, the load, the power and efficiency of each of them depend essentially on the individual load regimes of the pumps that work together. The cases of operation for the work of rotodynamic pumps in parallel are (Pardo and Ruiz, 1980, Martínez 2011):

Case 1: Pumps of equal hydraulic characteristics, operating against equal pipe systems to the point where the common impulse pipe (node) begins. In this case, the conditions set out above are met, that is, the bombs will be operating against the same load, therefore, it will be perfectly fulfilled that the expenses for equal loads are added.

Case 2: pumps with the same hydraulic characteristics, operating against different piping systems up to the point where the common discharge piping begins (node). For this particular case, since the piping systems are different up to the point where the secondary discharge pipes of the pumps join, it can be stated that these pumps are not operating against the same load, since, in short, they are operating against different systems.

Case 3: pumps of different hydraulic characteristics, operating against equal piping systems up to the point where the common rising main (node) begins. For this scenario, the piping systems are the same for each pump, but, since they are different pumps, the costs will also be different, and consequently, the losses that occur in each section up to the junction point will be different, therefore, the curves of the individual systems will be different, implying that the pumps are operating against different loads at the outlet of the same.

Case 4: pumps of different hydraulic characteristics, operating against different piping systems up to the point where the common rising main (node) begins. This is the so-called general case, for which it is evident, from all the above mentioned in cases 2 and 3, that the pumps will be operating against different loads.

The hydraulic simulation of pumping systems is particularly useful to evaluate the responses of such systems for certain operating conditions. One of the procedures used to carry out the simulation process is the well-known Virtual Pump Method. It is a simple and practical calculation procedure, recommended for those cases where there are pumps connected in parallel, when the installation conditions and/or hydraulic characteristics of the coupled pumps are not the same (Case 4, general case).

2 Development

2.1 Analytical treatment of the characteristic curves of parallel-coupled rotodynamic pumps

The equations modeling the three design characteristic curves of parallel coupled rotodynamic pumps for the case with equal hydraulic characteristics are (Turiño 1996, Martínez 2011):

Curva carga-capacidad (paralelo), (H_P-Q): $H_P = A \pm \frac{B}{n_b} Q - \frac{C}{n_b^2} Q^2$ (1)

where: H_P: load developed by the pumps coupled in parallel, (m); A: coefficient of the polynomial representative of the *H-Q* curve that defines the value of the load developed by the pump for zero flow or closed valve, (m); B and C: coefficients of the polynomial representative of the *H-Q* curve that depend on the pressure losses inside the pump, (s/m^2), (s^2/m^5) respectively; Q: flow rate driven by the combination of pumps in parallel, (m^3/s) and n_b: number of pumps working in parallel.

Curva potencia-capacidad (paralelo), (P_P-Q): $P_P = Dn_b \pm EQ \mp \frac{F}{n_b} Q^2$ (2)

where: P_P: power absorbed by the pumps working in parallel, (kW); D: coefficient of the polynomial representing the *P-Q*, curve, which defines the value of the power consumed by the pump for zero flow, (kW) and E and F: coefficients of the polynomial representing the *P-Q* curve dependent on the pump power losses, (s·kW/m^3), (s^2·kW/m^6) respectively.

Curva eficiencia-capacidad (paralelo), (η_P-Q): $\eta_P = \frac{G}{n_b} Q - \frac{H}{n_b^2} Q^2$ (3)

where: η_P: efficiency of the parallel pump combination, (add.) and G and H: coefficients of the polynomial representing the η-Q, curve, (s/m^3) and (s^2/m^6) respectively.

The $NPSH_r$ behavior of pumps does not change when they are placed in series or parallel, since by definition it is a variable that characterizes the suction capacity of a pump and is therefore independent of the coupling type.

Second-degree polynomials are usually proposed for all characteristic equations for rotodynamic pumps and pumps resulting from their coupling in parallel, in series, or in the case of a change in rotational speed and/or impeller diameter. In general, the order of the fitting polynomials based on the flow rate can be greater, thereby achieving improved goodnessof-fit, especially for mixed-type, axial, and specially designed pumps. Given these cases, equations 1-3 take the following form:

Load-capacity curve (parallel), (H_P-Q): $H_P = a_1 + \frac{a_2}{n_b} Q + \frac{a_3}{n_b^2} Q^2 + \ldots + \frac{a_n}{n_b^n} Q^n$ (4)

Power-capacity curve (parallel), (P_P-Q): $P_P = a_1 n_b + a_2 Q + \frac{a_3}{n_b} Q^2 + \ldots + \frac{a_n}{n_b^n} Q^{n+1}$ (5)

Efficiency-capacity curve (parallel), (η_P-Q): $\eta_P = \frac{a_1}{n_b} Q + \frac{a_2}{n_b^2} Q^2 + \ldots + \frac{a_n}{n_b^n} Q^n$ (6)

2.2 Hydraulic simulation of pumping systems. Pumped source systems

Hydraulic simulation of pumping systems in steady state (the most appropriate for the hydraulic analysis being performed) consists of obtaining the circulating flow rates through the pipes and the load at the nodes of the system using the system equilibrium equations (mass conservation equation, or continuity equation and the energy conservation equation) based on knowing: punctual consumption at the nodes (if any), the piezometric load at at least one node, and the relevant characteristics of the pipes (diameter, roughness and length) and the rest of the system elements (pumps, valves, accessories, etc.) (Cabrera 2009).

From the set of relationships above, two systems of nonlinear equations are obtained: one applying the continuity equation at nodes, and the other based on the pressure losses of the network elements. Their solution will yield the circulating flow rates and pressures at the nodes. The nonlinear nature of these systems of equations makes the application of numerical resolution methods essential. One of the most popular methods is the Newton-Raphson method, which finds the simultaneous solution to the system of mass and energy balance equations. The problem is solved by the iterative solution of a system of linear equations equal in size to the number of unknown piezometric loads.

The Newton-Raphson method is one of the many iterative methods available. Galguera (2015) uses the Node Iteration method to perform the hydraulic analysis of a well field pumping system. In this case, the model is first divided into independent subsystems, and then an iterative solution method similar to that used in the well-known Three-Tank Problem is applied. The methodology presented is superior to that presented by Miranda (2013), which applies this iterative method to simpler pumping source systems (fewer subsystems) with a Hazen-Willians friction coefficient, C, for the pipes, which is constant throughout the system.

Another of the most widely used numerical methods for successive iterations, primarily due to its ease of programming and use in simulation software such as EPANET, is the Gradient Method, proposed in 1987 by Todini and Pilati. It combines techniques based on optimization methods with techniques based on the nodal Newton-Raphson method. It begins by applying optimization techniques, which guarantee the existence and uniqueness of the solution by minimizing the objective function. These are essential conditions for subsequent convergence when using the Newton-Raphson method. The problem is finally driven to an algebraic solution through the iterative process known as Incomplete Choleski Factorization/Modified Conjugate Gradient (ICF/MCG).

The Virtual Pump Method, on the other hand, has not been the subject of much research in recent years, despite being a simpler procedure than the iterative methods mentioned above. The concept of virtual pumps simplifies the hydraulic calculation of actual operating points and, in general, the physical understanding of the problem. If, for a given pumping system, the characteristic curve of the virtual pump is taken to include both the suction and discharge pipes up to the discharge point, the length of the system against which pumping would be required would be zero, and the corresponding head losses would be zero. Consequently, the characteristic curve of this system would be given simply by the values of static height versus flow rate (Galvis and Castilla 1993).

2.3 Virtual pump method

In everyday practice, various procedures have been used to determine the operating points of each of the pumps operating, either individually or in parallel, and discharging into the same main drive system. These procedures can be summarized as follows:

(1) The pressure losses in the suction and secondary discharge of each pump (excluding the main discharge pipe or system) are ignored when calculating the piping system's characteristic curve.

(2) Any differences in pressure losses that may exist in the secondary discharge and suction of the pumps are ignored, and any pressure losses considered significant in this part of the installation are included when calculating the main discharge piping system's characteristic curve.

Since in the two previous criteria the losses in the duct system part of each machine are considered equal, it follows that, if the pumps work in parallel and lift from the same level in the suction, they must necessarily work against the same total load. It should be noted that calculation procedures based on the above criteria, particularly in the second one, produce good results for some practical problems, especially when the head losses in the ducts of each pump are small compared to those that occur in the main discharge pipe and therefore do not determine the shape of the characteristic curve of the system against which it is pumped (Castilla and Galvis 1993).

Conceptually, virtual pumps are imaginary machines that lack secondary suction and discharge piping. These pumps consist of the actual pump supplied by the manufacturer and all accessories and piping up to the point of connection with the main discharge piping. These devices are characterized by always working against the same total load when operating in parallel, without requiring assumptions that might imply only approximate solutions, as would be the case with the previous criteria. This concept of virtual pumps greatly simplifies the hydraulic calculation of actual operating points and, in general, the physical understanding of the problem (Castilla and Galvis 1993).

If, for a given pumping system, the virtual pump's characteristic curve is taken to include both the suction and discharge piping up to the discharge point, the length of the system against which pumping would be carried out would be zero, and the corresponding head losses would be zero. Consequently, the characteristic curve for this system would be given simply by the head versus flow rate values (Castilla and Galvis 1993).

Technically speaking, virtual pumps are analogous in their hydraulic behavior to the installation of a diaphragm between the discharge flange of a rotodynamic pump and the discharge pipe flange. It should be noted that this constitutes a pure throttling, the losses of which directly affect the pump's head-capacity characteristic curve. As is known, pressure losses (head losses) due to the diaphragm follow a quadratic or parabolic curve. Therefore, the new pump characteristic curve, when the calibrated diaphragm is installed, differs from the previous curve at all points due to the pressure drop.

This method can be used for any of the four parallel pump design schemes; it is especially recommended for Case 4 (the general case). As noted, this is an alternative calculation methodology to numerical methods for simulating pumping systems, specifically for pumped source systems. Hydraulically, it is a procedure for addressing the problem of operating rotodynamic pumps operating in parallel in branched pipelines with independent and common pipe sections.

2.4 Approach to the virtual pump method

Using Figure 1 as a reference, Bernoulli's equation is applied from the water levels in the suction tanks of each pump to point A (the common or central node of the pumping source system), where there is a common head H_A (piezometric head at node A) for the connections between pumping stations B1 and B2 and node A. In this case, the flow rates of the individual pumps are summed at this node, the beginning of the common section. This means that the total pump head values must be equal at this point (SIHI 2003).

The total load at node A is obtained from the *Hvs.Q* characteristic curves of each individual pump, reducing them according to the system curves of each individual system (non-common sections). For system B_1-A, the following is obtained:

$H_1 \pm Z_1 = H_A + h_{fB1-A}$ (7)

While on the B_2-A branch:

$H_2 \pm Z_2 = H_A + h_{fB2-A}$ (8)

where: H_1: total head of pump 1, (m); H_2: total head of pump 2, (m); H_A: head at node A, (m); Z_1: height of the water level in the suction tank of pump station 1, with respect to an established reference plane, (m); Z_2: height of the water level in the suction tank of pump station 2, with respect to an established reference plane, (m); hf_{B1-A}: head losses along branch B_1-A, (m) and hf_{B2-A}: head losses along line B_2-A (m).

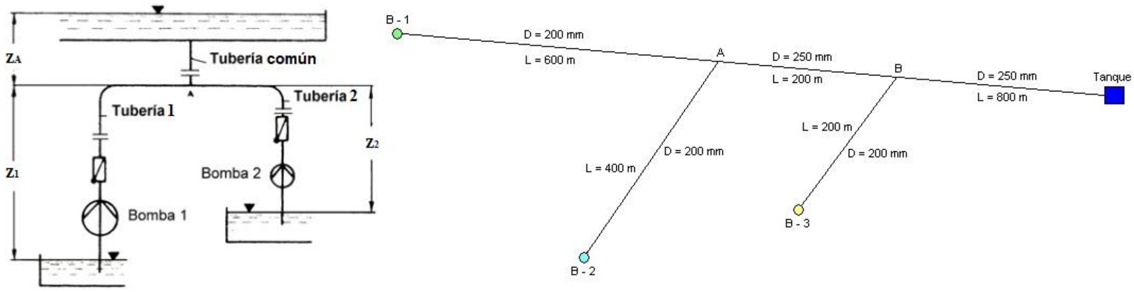

Figure 1. Operation of rotodynamic pumps with different hydraulic characteristics in parallel with independent and shared pipeline sections (Case 4) (left image). Example of an operating diagram of a pumped source system for a well field (right image)

Equations 7 and 8 provide the theoretical basis for analyzing the behavior of pumps operating in parallel. In their general form and for any number of pumps connected to the same node (node A in this example), these equations transform to obtain the general equation for a virtual pump B_{nA} (Equation 9). This transformation process can be seen graphically in Figure 2.

$H_A = H_n \pm Z_n - h_{fBn-A} = H_{nA}$ (9)

These virtual or reduced head-capacity curves can be coupled in parallel, as shown in Figure 2 (right image), by summing the flow rates at the same total height. The intersection of this H_{Pr}-Q curve with the common piping system curve (pipe 3 in the left image of Figure 1) gives the total circulation flow rate.

The individual pump flow rates are obtained by intersecting the horizontal line (right image in Figure 2) with the individual reduced characteristic curves. These flow rates can then be used to obtain the head (reduced and actual), power, efficiency, and $NPSH_r$ values for each pump, respectively.

The application of the Virtual Pump Method in a pumped source system is detailed below:

(1) Determine (graphically or analytically) the actual load of the n pumps for different reference flow rates.

(2) Perform dynamic correction (reduction of the actual pump load due to pressure losses in the piping of each pump's individual system) for each pump. This can also be done graphically or analytically.

(3) Calculate the static correction, which is obtained from the difference between the water levels in each suction tank and the virtual level established for the system, Z_{sv}. The latter could be calculated in several ways, all of them valid: as an average of the water levels in all the suction tanks in the pumping system, by setting a fixed value for Z_{sv}, which could be the smallest value of all the levels, the largest, or the one decided to establish as the standard for the n pumps. The

disadvantage of the first option is that it would be necessary to compute n static corrections, while, for the other three variants, the number of static corrections to be performed would be n-1.

(4) Compute for each pump, the virtual load corresponding to each reference flow according to the following expression:

$$H_{nN} = H_n - hf_n - (Z_n - Z_{sv}) \qquad (10)$$

Where: H_{nN}: virtual head of pump n, (m); H_n: actual head of pump n, (m); hf_n: head losses in the individual system of pump n, (m); Z_n: height of the water level in the suction tank of pump station n, (m) and Z_{sv}: height of the virtual level set for the pumping system, (m)

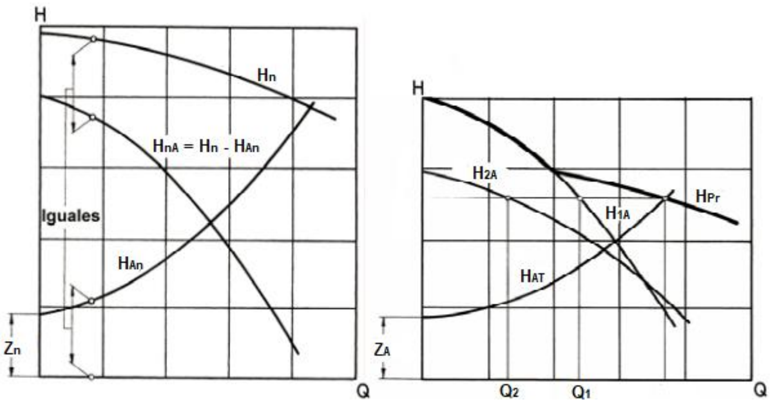

Figure 2. Reduced characteristic curve of a pump n at point A (left). Characteristic curve for the parallel combination of two virtual pumps operating against a common pipeline (right).

(5) Obtain (graphically or analytically) the load-capacity characteristic curves of the virtual pumps B_{nN} at the node.

(6) Combine (graphically or analytically) the *n* virtual pumps in parallel to obtain the virtual equivalent pump, B_{1-n}, at the node.

Conceptually, once this last step has been completed, there would be a virtual pumping station in the node (example: node A for Figure 1 (image on the left)), where the n virtual pumps will be installed with their virtual equivalent in parallel, whose water level in the suction tank will be the virtual level established for the system, Z_{sv}.

Using the left image in Figure 1 as an example, the pumping system would be simplified to a virtual pumping station-pipe-discharge tank scheme. The system could be further restructured by dynamically correcting the virtual pump combination in parallel, B_{1-n}, from the common pipe. In theory, this would be equivalent to having a pumping station with a virtual pump, B_{1-n-T}, coupled to the suction tank or reservoir (common pipe length equal to zero).

For a better understanding of the methodological procedure for applying the Virtual Pump Method, the simplest scheme that can be proposed will be applied to an example of a pumped source system such as the one illustrated in the image on the left of Figure 1.

2.5 Application example for a multi-node pumped source system (well field)

Pumps 1 and 2 used in this study are single-stage, deep-well submersible pumps (Worthington Pumps, 1986). High-density polyethylene (HDPE) will be used as the piping material, with an absolute pipe roughness of ϵ = 0.0000025 m. A kinematic viscosity of water of υ = 10-6 m2/s will be assumed. The water levels in the suction tanks are 10 m and 20 m for pumps 1 and 2, respectively. The internal diameter of both pipes is 200 mm. The equivalent pipe lengths are 600 m, 400 m, and 1,000 m for branches 1 and 2 and the common or collector pipe of the system, respectively.

Applying steps 1 and 2 described for the procedure for working with the Virtual Pump Method, the results shown in Table 1 and Figures 3 and 4 are obtained.

Table 1. Data from the characteristic curves of real and virtual pumps

Flow rate	Operating ranges of real pumps		Head losses in the individual piping system of each pump		Virtual pump loads	
Q (m^3/h)	H_1 (m)	H_2 (m)	h_{f1} (m)	h_{f2} (m)	H_{1A} (m)	H_{2A} (m)
0	46.80	28.91	0	0	46.800	28.910
20	46.77	28.86	0.108	0.072	46.662	28.788
40	45.64	28.27	0.369	0.246	45.271	28.024
60	43.38	27.14	0.762	0.508	42.618	26.632
72	41.48	26.21	1.058	0.705	40.421	25.504
84	39.19	25.08	1.397	0.931	37.793	24.149
96	36.49	23.75	1.778	1.185	34.712	22.565
108	33.4	22.23	2.200	1.467	31.200	20.763
120	29.89	20.53	2.663	1.775	27.227	18.755
132	25.98	18.62	3.165	2.110	22.815	16.510
144	21.68	16.52	3.707	2.471	17.973	14.049
150	19.37	–	3.992	–	15.378	–

In step 3, the static correction for the system is determined. For this, it must first be assumed what the virtual level, Z_{sv}, to be established for the system will be. Of the possible variants, it is proposed that it be the average of both water levels in the two suction reservoirs, i.e. 15 m. Therefore, the static corrections would be calculated as: ($Z_1 - Z_{sv} = 10 - 15 = -5$ m) y ($Z_2 - Z_{sv} = 20 - 15 = 5$ m) for pumps 1 and 2 respectively.

The calculated values of the virtual loads using equation 10 in step 4 are shown in Table 1. As part of step 5, the graphs of the virtual load-capacity curves for pumps 1 and 2 are illustrated in Figures 3 and 4, along with their representative polynomials. These figures also show the graphs of the actual Hvs.Q curves and the head loss curves for each system independent of each pump, along with their respective modeling equations.

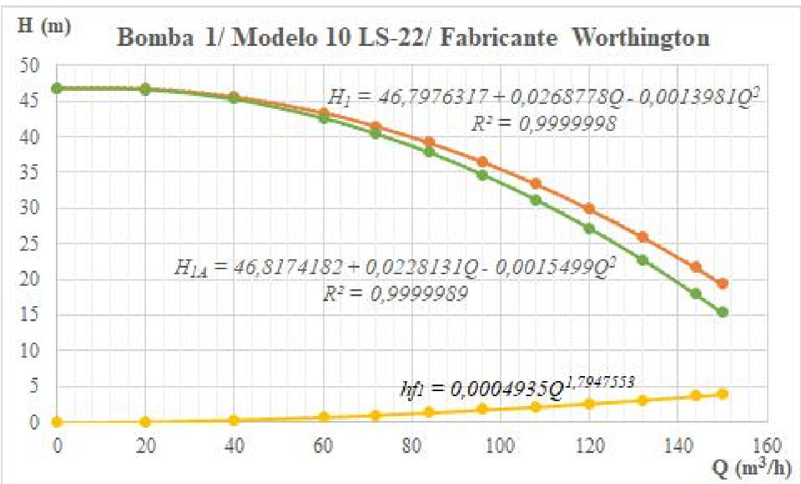

Figure 3. Graphical and analytical expressions of the load vs. real and virtual capacity curves of pump 1 together with the pressure losses for its piping system.

For the development of step 6, the following graphic and analytical paths are proposed:

Graphical Solution

The graphical solution to the problem of two pumps 1 and 2 operating in parallel is obtained by constructing the curve for the two virtual pumps operating in parallel, as follows:

(1) Virtual pump B_{1A} produces a flow rate Q_1 for the load H_A (load at node A). Virtual pump B_{2A} provides a flow rate Q_2 for the same load H_A. Both virtual pumps, working together against the load H_A, produce the flow rate $Q_A = Q_1 + Q_2 = Q_P$. The point on the curve for working together is represented by the coordinates: $H = H_A$ and $Q = Q_A = Q_1 + Q_2$.

(2) For different values of H_A, the H_A vs. Q_A, indicated in Figure 5 as the $B_{1\text{-}2A}$ coordinate, corresponds to a virtual pump that replaces the entire system up to point A.

(3) The intersection point of this curve with the characteristic curve of the piping system for the common section A-T (node-discharge tank), excluding the secondary suction and discharge pipes, is the solution sought to the problem, i.e., the actual operating point for this system. The data for the characteristic curve of the piping system for the A-T section are shown in Table 2.

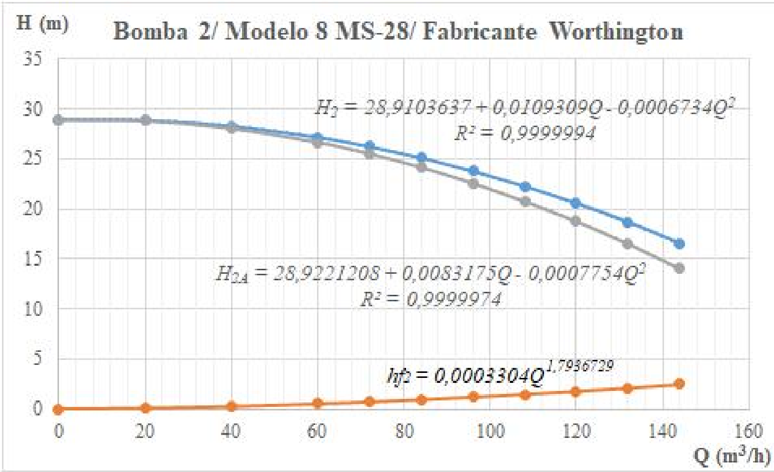

Figure 4. Graphical and analytical expressions of the load vs. real and virtual capacity curves of pump 2 together with the pressure losses for its piping system.

Table 2. Data of the characteristic curve of the common drive pipe for the A-T section

Expense	Losses in the main discharge pipe A-T	Piping system load
Q (m^3/h)	h_{fc} (m)	$H_{sist.}$(m)
0	0.000	15.000
36	0.175	15.175
72	0.604	15.604
108	1.253	16.253
144	2.107	17.107
180	3.158	18.158
216	4.398	19.398
252	5.823	20.823
288	7.430	22.430
324	9.214	24.214

From the graph in Figure 5, it can be seen that the operating flow rate of the system is 247 m^3/h for a workload of 20.6 m. For this load, the flow rates of each pump are: Q_1 = 137 m^3/h and Q_2 = 110 m^3/h.

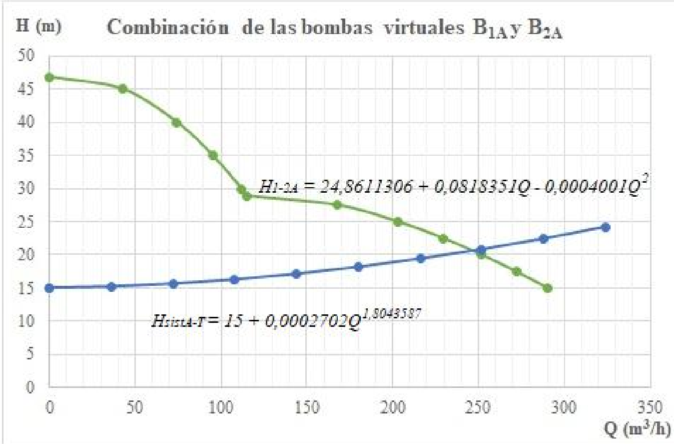

Figure 5. Operation of virtual pumps 1 and 2 in parallel against the common A-T pipe system

Analytical solution

The equations of the head-capacity curves of the original pumps, their respective virtual pumps, the characteristic curve of the A-T pipeline system (common section), and the curve of the parallel combination of the two virtual pumps operating against the collector pipeline system are shown in Figures 3, 4, and 5, in that order. For the case of this last formulation, it is only applicable within the interval where the flow contributions of the two virtual pumps join the A-T system.

Working with the analytical expressions, the following solutions are obtained: operating point for the A-T system: H_A= 20.624 m and Q_A= Q_P = Q_1+ Q_2 = 247.348 m^3/h; for this value of H_A, the values of the flow rates supplied by each of the pumps are obtained: Q_{1A} = Q_1 = 137,599 m^3/h and Q_{2A} = Q_2 = 109,749 m^3/h.

Generalizing this procedure to a set of n pumps coupled in a multi-node parallel interconnected system (e.g., a well field as illustrated in Figure 1 (image on the right)), the procedure would be as follows: if a third pump were coupled, whose secondary drive is connected at node B in the aforementioned figure:

(1) Curve $B_{1,2A}$ is transformed into a second virtual curve by subtracting the pressure losses in section A-B from each load for each flow rate Q_{1A} + Q_{2A}, obtaining curve $B_{1,2B}$.

(2) In a similar manner to how the remaining virtual curves for pumps 1 and 2 were obtained, the curve for virtual pump B_{3B} is obtained.

(3) The two previous curves are combined to obtain curve $B_{1,2,3B}$.

(4) The intersection of this curve with the characteristic curve of the piping system for section B-T is found, a confluence that defines the operating point for the three pumps 1, 2, and 3 operating together in parallel.

(5) As a final step, the values of H_B and Q_B are obtained, which define the coordinate of the operating point for the case of three pumps operating in parallel.

3 Conclusion

The virtual pump method is a practical and simple procedure used to obtain fitted characteristic curves for parallel pump combinations. This concept of virtual pumps greatly simplifies the hydraulic calculation of real operating points and, in general, the physical understanding of the problem. This paper provides an introduction to the method and its fundamental hydraulic principles.

The procedure can be summarized as follows: Using the information from the pump characteristic curves and the piping system curves from the pumps to the piping system junction point, the latter are subtracted from the pumps' load-capacity curves, thus obtaining the fitted pump characteristic curves for the load corresponding to the piping system junction point. Once this first step has been completed, the conventional method of summing the fitted curves can be applied, since they all refer to the same load.

A practical example of two pumps coupled in parallel is presented to illustrate this method more efficiently, showing its advantages and practical utility in simulating the operation of pumping systems.

Conflicts of interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Bombas Worthington (1986). "Catálogo técnico: bombas sumergibles", Sucursal Madrid, Madrid

[2] Cabrera E. (2009). "Ingeniería hidráulica aplicada a los sistemas de distribución de agua", Editorial Unidad Docente Mecánica de Fluidos, Universidad Politécnica de Valencia, t. 1 y 2, 3ra edición, ISBN 8461339495, Valencia, España.

[3] Castilla A. y Galvis G (1993). "Bombas y estaciones de bombeo", Instituto de Investigación y Desarrollo en Abastacimiento de Agua, Saneamiento Ambiental y Conservación del Recurso Hídrico (CINARA)-Universidad del Valle, Ed. Ultragraf Editores, Cali, Colombia.

[4] Cherkasski V. M. (1986). "Bombas ventiladores compresores", Ed. Mir Moscú, Unión de Repúblicas Socialistas Soviéticas (URSS).

[5] Galguera C. L. (2015). "Metodología para el diseño y simulación hidráulica de un sistema fuente por bombeo desde campo de pozos", Trabajo de diploma, Facultad de Ingeniería Civil, Instituto Superior Politécnico "José Antonio Echeverría", Cujae, La Habana, Cuba.

[6] Karassik I., Krutzsch W., Fraser W. and Messina J. (1986). ″Pump Handbook", 2nd edition, Ed. John Wiley & Sons, Inc., ISBN 13: 978-007-03-3302-4, New York, United States, 1986.

[7] Martínez Y. (2011). "Metodología para el diseño hidráulico de las estaciones de bombeo para acueducto", Tesis de doctorado, Instituto Superior Politécnico "José Antonio Echeverría" (Cujae), La Habana, Cuba.

[8] Martínez Y. y Riaño F. (2010). "Características peculiares de la operación de bombas rotodinámicas en paralelo", Ciencias Técnicas Agropecuarias, 19 (2): 38-43, Universidad Agraria de La Habana Fructuoso Rodríguez (UNAH), ISSN 1010-2760, Mayabeque, Cuba.

[9] Miranda Y. (2013). "Metodología para el diseño hidráulico de las estaciones de bombeo en función de la velocidad específica", Trabajo de diploma, Facultad de Ingeniería Civil, Instituto Superior Politécnico "José Antonio Echeverría", Cujae, La Habana, Cuba.

[10] Pardo R. y Ruiz M. I. (1980). "Algunas consideraciones sobre el funcionamiento de bombas en paralelo", *Revista Ingeniería Hidráulica*, 1 (1): 20-28, Instituto Superior Politécnico Superior "José Antonio Echeverría", La Habana, Cuba.

[11] Pérez Franco D. (2012). "Estaciones de Bombeo", Editorial Félix Varela, ISBN 978-959-07-1379-8, La Habana, Cuba.

[12] Sterling SIHI (2003). "Principios Básicos para el Diseño de Instalaciones de Bombas Centrífugas", (Manual técnico), 7th edition, Ed. Sterling Fluid Systems Group, Madrid, España, 2003.

[13] Turiño, I. M. (1996). "Procedimientos metodológicos para el diagnóstico operacional en sistemas de bombeo mediante modelos matemáticos", Tesis de doctorado, Facultad de Ingeniería Mecánica, Universidad Central de las Villas Marta Abreu (UCLV), Santa Clara, Cuba.

Author Contributions

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He participated in the development of the methodological procedure for applying the Virtual Pump Method, as well as in the processing of the case study data, contributing to its analysis and interpretation. He participated in the search for bibliographic references and in the writing of the paper.

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He contributed to the research design and analysis of the case study results, as well as in the search for bibliographic information, review, and final writing of the paper.