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Hydraulic simulation of pumped water source systems through well fields: Part 1

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Abstract: In applying different simulation analysis models to hydraulic systems, certain simplifications and specific features allow for simpler models that require less information and less sophisticated mathematical techniques, and therefore fewer computational resources. In this case, we find the Three Tanks iterative solution model, applied in this work to pumped water source systems within a well field topology (a multinodal interconnected branched system). This article, the first of three on this line of research, develops the Nodal Iteration Method, an iterative solution procedure that offers an analysis approach based on the component subsystems of the overall well field hydraulic system. This solution procedure focuses on determining the piezometric head values at the nodes that guarantee the balance of the overall hydraulic system.
Key words: simulation; hydraulics; source systems; pumping; field wells

1 Introduction

The term hydraulic simulation generally refers to the process of imitating the behavior of a system using a mathematical representation of the real system, called a model. Simulations of hydraulic systems, which reproduce the dynamics of an existing or proposed system, are typically performed when it is impractical to conduct tests on the actual system or to evaluate a system before construction.

Hydraulic network simulations can be used to predict the response of these systems to events under a wide range of conditions without disrupting the actual system. Simulations allow for anticipating problems in proposed or existing systems and evaluating solutions before investing time, money, and materials in a real project. Regardless of the project, model-based simulation can provide valuable information that helps specialists make well-informed decisions (Haestad et al., 2003).

Water source systems, such as hydraulic networks, can be classified as either pumped or gravity-fed, and these, in turn, constitute a subsystem within water supply systems. The difference between the two lies in the inclusion, in the case of pumped systems, of the pumping station(s) as a component. The different design and operation schemes for water source systems will depend on the characteristics of the suction and discharge intakes and whether the demand is continuous (constant flow) or variable over time (variable flow) (Martínez 2011).

The main design schemes of the most common pumped source systems in practice can be grouped into the following cases (Martínez 2011):

Case 1. Source → Pumping Station → Carrier Pipe → Regulating Tank (free inlet and forced discharge with a fixed delivery head). Examples of such schemes that are widely used in practice are:

· Storage Tank (Cistern) of the Water Treatment Plant → Pumping Station → Main Pipeline → Regulating Tank (Free or Submerged Inlet with Fixed Delivery Head).

· Reservoir → Pumping Station (Tail of the Dam) → Main Pipeline → Receiver Tank of the Water Treatment Plant (Free Inlet with Fixed or Variable Delivery Head).

· Canal → Pumping Station → Main Pipeline → Receiver Tank of the Water Treatment Plant (Submerged or Free Inlet).

· Wellfield (Including Pumping Stations and Secondary Pipelines/Tubes) → Main Pipeline → Receiver Tank of the Water Treatment Plant (Submerged or Free Inlet).

· Wellfield (Including Pumping Stations and Secondary Pipelines/Tubes) → Main Pipeline → Regulating Tank.

These types of systems are characterized by drawing water from both groundwater and surface water sources, which is why the final receiving structure is distinguished according to each specific case. Another characteristic of these systems is that the pumping service is continuous and the demand is constant, resulting in a fixed design flow rate for the system.

Case 2. Source → Pumping Station → Pipeline → Delivery Node to the Distribution Network. Practical examples of this scheme are as follows:

Storage Tank (Cistern) of the Water Treatment Plant → Pumping Station → Main Pipeline → Distribution Network Node.

Storage Tank (Cistern) → Pumping Station → Main Pipeline → Distribution Network Node.

In these types of cases, information is available on the consumption evolution curve of the network for the typical design day, established for the delivery node to the network, where the maximum flow rate and the minimum load required at the delivery node in the network for that maximum flow rate are known.

Pumped water supply systems can be classified as simple or complex, based on their topological configuration, from the water source to the delivery node to the network. A pumped water supply system that includes only one of the configurations described in the cases discussed is considered a simple system. A complex pumped water supply system will be composed of several simple subsystems and, therefore, is classified numerically, where the sections or simple subsystems are numbered from upstream to downstream.

One complex design scheme encountered in practice, albeit on a significantly reduced scale, consists of pumping booster systems. These are classified into direct and indirect booster systems. Direct booster operation operates as a typical series pumping system, following the scheme: Pumping Station → Pumping Station; while indirect boosting is characterized by the scheme: Pumping Station → Regulating Tank (Free or Forced Inlet) → Pumping Station.

Hydraulic simulation of pumping systems is especially useful for evaluating the responses of these systems under specific operating conditions. Several procedures are used to perform simulations on pumped water source systems, specifically in well fields. These can be grouped into several categories (Fuertes et al., 2002; Galguera, 2015): methods based on the Newton-Raphson method; methods based on Hardy-Cross theory; methods based on linear theory; methods based on topological network formulations (Gradient Method); the Virtual Pump method; the node iteration method; and others. This work will focus on developing the last of these methods by implementing a common practical example used in engineering practice.

2 Development

2.1 Hydraulic simulation of pumped source systems

Simulations of pumped water supply systems can be steady-state or extended-period. Steady-state simulations represent a snapshot in time and are used to determine the operating behavior of a system under static conditions. This type of analysis can be useful, for example, to determine the short-term effect of fire flows or average demand conditions on a system. Extended-period simulations (EPS) are used to evaluate system performance over time. This type of analysis allows the user to model the filling and emptying of tanks, the opening and closing of valves, and the pressures and flows that change throughout the system in response to varying demand conditions and automatic control strategies formulated by the modeler (Fuertes et al., 2002).

The hydraulic simulation of pumped source systems in steady state (the most suitable for the hydraulic analysis carried out) consists of obtaining the flow rates circulating through the pipes and the head at the nodes of the system using the system equilibrium equations (mass conservation equation, or continuity equation and the energy conservation equation), starting from known data: point consumption at the nodes (if any), the piezometric head at at least one node, and the relevant characteristics of the pipes (diameter, roughness and length) and the rest of the elements of the system (pumps, valves, accessories, etc.) (Cabrera 2009).

From the set of relationships described above, two systems of nonlinear equations are obtained. One is derived by applying the continuity equation at the nodes, and the other from the head losses of the network elements. Solving these systems will yield the flow rates and the pressures or heads at the nodes. The nonlinear nature of these systems of equations makes the application of numerical methods indispensable. One of the best methods is the Newton-Raphson method for finding the simultaneous solution of the system of mass and energy balance equations. The problem is solved by iteratively solving a system of linear equations whose size is equal to the number of unknown piezometric heads.

The Newton-Raphson method is one of many iterative methods for solving this problem. Galguera (2015) uses the Nodal Iteration Method to perform the hydraulic analysis of a pumped source system in a well field. In this case, the model is first divided into independent subsystems, and then an iterative solution method similar to that used in the well-known Three Tank Problem is applied. The methodology presented here surpasses that of Miranda (2013), which applies this iterative method to simpler pumped source systems (fewer subsystems) and with a constant Hazen-Williams friction coefficient, C, for the pipes throughout the system.

Another widely used numerical method of successive iterations, primarily due to its ease of programming and use in simulation software such as EPANET, is the Gradient Method, proposed in 1987 by Todini and Pilati. It combines techniques based on optimization methods with techniques based on the nodal Newton-Raphson method. It begins by applying optimization techniques, which guarantee the existence and uniqueness of the solution by minimizing the objective function—essential conditions for subsequent convergence when using Newton-Raphson techniques. The problem is finally led to an algebraic solution through the iterative process known as the Incomplete Choleski Factorization/Modified Conjugate Gradient Algorithm (ICF/MCG).

In this first paper, the Nodal Iteration Method will be analyzed as an application of a generalized solution to a branched pipe problem, but in reverse to its classical conception. The hydraulic analysis of the system as a whole and its independent subsystems is performed by solving the Three Tank Problem. This method is simpler in its calculation algorithm than the Virtual Pump Method, but it involves a lengthy process of iterative sequences to find the solution. Once the solution is found, the flow rates or velocities in the pipes of the hydraulic system, the respective hydraulic variables of the pumps, and the values of the pressures or piezometric heads at the nodes are obtained.

On the other hand, the Virtual Pump Method has not been the subject of much research in recent years, despite being a simpler and more practical procedure than the iterative methods mentioned above. The concept of virtual pumps simplifies the hydraulic calculation of the actual operating points and, in general, the physical understanding of the problem. If, for a given pumping system, the characteristic curve of the virtual pump is taken to include both the suction and discharge pipes up to the discharge point, the length of the system against which pumping would occur would be zero, and the corresponding head losses would be negligible. Consequently, the characteristic curve of this system will be given simply by the values of static head versus flow rate (Galvis and Castilla 1993).

2.2 Hydraulic simulation of the operation of rotodynamic pumps in pumped source systems

Of the four scenarios involving the operation of rotodynamic pumps in parallel within pumped storage systems, the most common is one in which pumps with different hydraulic characteristics operate against separate pipe systems up to the point where the common discharge pipe begins (node). This is known as the general case, in which it is clear that the pumps will be operating against different loads and delivering different flow rates (Pardo and Ruiz, 1980):

The equations that model the three characteristic design curves of rotodynamic pumps coupled in parallel for the case with equal hydraulic characteristics are (Turiño 1996, Martínez and Riaño 2010):

Load-capacity curve (parallel) *(HP-Q)*: $H_P = A \pm \frac{B}{n_b}Q - \frac{C}{n_b^2}Q^2$ (1)

where: H_P: head developed by the pumps coupled in parallel, (m); A: coefficient of the polynomial representing the *H-Q* curve that defines the value of the head developed by the pump for zero flow or closed valve, (m); *B* and *C*: coefficients of the polynomial representing the *H-Q* curve that are a function of the head losses inside the pump, (s/m2), (s2/m^5) respectively; *Q*: flow rate driven by the combination of the pumps in parallel, (m^3/s), and nb: number of pumps working in parallel.

Power-capacity curve (parallel), *(P_P-Q)*: $P_P = Dn_b \pm EQ \mp \frac{F}{n_b}Q^2$ (2)

where: P_P: power absorbed by the pumps working in parallel, (kW); *D*: coefficient of the polynomial representing the *P-Q* curve, which defines the value of the power consumed by the pump for zero flow, (kW), and *E* and *F*: coefficients of the polynomial representing the *P-Q* curve dependent on the power losses of the pump, (s·kW/m^3), (s^2·kW/m^6) respectively.

Efficiency-capacity curve (parallel), *(ηP-Q)*: $\eta_P = \frac{G}{n_b}Q - \frac{H}{n_b^2}Q^2$ (3)

where: η_P: efficiency of the combination of pumps in parallel, (dimm.), and *G* and *H*: coefficients of the polynomial representing the *η-Q* curve, (s/m^3), (s^2/m^6) respectively.

The *NPSH*r behavior of pumps does not change when they are placed in series or in parallel, since by definition it is a variable that characterizes the suction capacity of a pump and is therefore independent of the type of coupling.

3 Iteration method on the node

The conceptual basis of the node iteration method is founded on the solution to the Three Tank Problem, which can be solved using various iterative techniques. A review of the extensive literature on this topic has revealed the absence of a procedure for branched systems of the type composed of well fields (multinodal interconnected branched systems). The classic problem, in its most common cases, is approached from the perspective of three tanks, and less frequently, pumps and tanks. For any of these variants, the iterative solution is the most frequently used, and in isolated cases, an analytical solution has been outlined that allows for the resolution of the following two cases:

1) Determining flow rates, given the network characteristics and the elevation differences between the reservoirs; therefore, working with three equations with three unknowns.

2) Determining the elevation difference between the reservoirs, given the flow rates and the network characteristics; therefore, there will be two equations with two unknowns, since the flow rate equations are an identity.

Figure 1 will be used as an illustrative support to explain the general methodology for the hydraulic simulation of the field model of wells with continuous demand and constant flow rate.

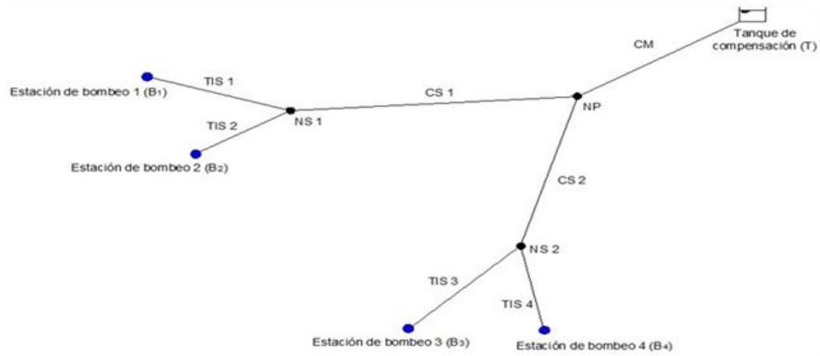

Figure 1. Plan view of a pumped water source system consisting of a well field (Ref. Martínez and Riaño, 2010)

When referring to a main pipeline and other secondary pipelines, direct reference is made to the design scheme consisting of an interconnected system, such as a well field, for example, as shown in Figure 1. In such schemes, the principal pipeline (*CP)* or main pipeline (*CM*) connects the central node (*NC*) or principal node (*NP*) of the well field to the delivery node of the network or to the delivery point of the regulating or compensation tank (T). Meanwhile, the secondary pipelines (*CS*) link the secondary nodes (*NS*) of the system to each other or to the principal node, depending on the scheme being analyzed. The secondary pumping pipelines (*TIS*) are those that connect the pumping stations (*B*) at each well to their respective secondary nodes.

Before applying this method, it will be necessary to divide the analysis model into the required number of independent subsystems. Using Figure 1 as an example, the overall system can be broken down into three subsystems, namely:

- Subsystem 1: Composed of pumping stations 1 and 2 (B_1 and B_2), secondary pressure pipes 1 and 2 (*TIS 1* and *TIS 2*), secondary node 1, NS_1, and secondary pipeline 1, *CS 1*, to the main node, *NP*.
- Subsystem 2: Composed of pumping stations 3 and 4 (B_3 and B_4), secondary pressure pipes 3 and 4 (TIS_3 and TIS_4), secondary node 2, NS_2, and secondary pipeline 2, CS_2, to the main node, *NP*.
- Subsystem 3: Composed of the junction node of secondary pipelines 1 and 2 respectively, *NP*, the main pipeline, *CM* or *CP*, and the discharge tank, *T*.

The results to be obtained from the simulation process for the operational model are the flow rates contributed by each pumping station to the discharge reservoir, the flow velocities and the loads at the nodes. In this case, the analysis is carried out from the boundaries of subsystems 1 and 2 towards the main node, independently for each system. The analysis uses the energy conservation equation (Bernoulli's equation), performing a flow balance (mass conservation) at the relevant nodes, namely NS_1, NS_2 and *NP*, and checking the mass balance at each of them.

Although the example in Figure 1 has been used as an explanatory reference for the methodology presented, these types of systems can have diverse topological forms that vary in complexity. The model illustrated here is one of the simplest, but the methodological procedure is applicable to any topology, no matter how complex. In this sense, and depending on the number of nodes and conductors derived from them, there can be primary nodes (main or central node), secondary nodes, tertiary nodes, quaternary nodes, quinary nodes, etc. The conductors will be classified according to the nodes they connect. The order in the classification of nodes and conductors will increase from downstream (discharge reservoir) towards the pumping stations. Therefore, a general classification is proposed for these pumped water systems interconnected by pumping stations, analogous to the numerical hydrological classification of rivers.

The proposed procedure is structured as follows:

1st step. Assume values of piezometric levels at the main node, CP_{NP}, and at the secondary nodes that feed into it, $CP_{NS}n$.

2nd step. Calculate the pressure losses in the secondary pipelines, hf_{CSn}.

$$CP_{NSn} = CP_{NP} + hf_{CSn} \rightarrow hf_{CSn} = CP_{NSn} - CP_{NP} \qquad (4)$$

Where: CP_{NSn}: piezometric head at the secondary nodes, (m); CP_{NP}: piezometric head at the main node, (m), and hf_{CSn}: pressure losses in the secondary conductors, (m).

Step 3. Determine the flow rates through the secondary pipes, Q_{CSn}.

For this, we will use expression (5), which is the modified Darcy-Weisbach formulation based on the flow rate:

$$hf_{CSn} = K_{CSn}Q^2_{CSn} = 8.26 \cdot 10^{-2} \cdot f_{supCSn}\frac{L_{CSn}}{D^5_{CSn}}Q^2_{CSn} \rightarrow Q_{CSn} = \sqrt{\frac{hf_{CSn}D^5_{CSn}}{8.26 \cdot 10^{-2} \cdot f_{supCSn}L_{CSn}}} \qquad (5)$$

Where: K_{CSn}: characteristic coefficient of the conduction, (s^2/m^5); L_{CSn}: length of the secondary conductor n, (m), and D_{CSn}: diameter of the secondary conductor n, (m).

Initial assumed values for the friction coefficients in the secondary conductors, f_{supCSn}, will be used to determine the flow rate using the aforementioned equation. Preliminary values of 0.02 are usually used. Then, as part of an iterative calculation cycle, the actual values of the friction coefficients of the secondary conductors, $f_{realCSn}$, are calculated from the obtained flow rates, using the Swamee-Jain formula within its range of validity.

$$f_{realCSn} = 0.25\left(log(\frac{\varepsilon}{3.7D_{CSn}} + \frac{5.74}{N^{0.9}_{R\ CSn}})\right)^{-2} \qquad (6)$$

Where: ε: absolute roughness of pipe n, (m), and N_{RCSn}: Reynolds number of the secondary conductor n for the flow rate obtained in the preceding step, (dimm.).

$$N_{RCSn} = \frac{V_{CSn}\,D_{CSn}}{\nu} = \frac{4Q_{CSn}}{\pi D_{CSn\nu}} \qquad (7)$$

Where: V_{CSn}: average circulation speed in the secondary conductor n (m/s), and ν: kinematic viscosity, (m^2/s).

It must be verified that $f_{supCSn} = f_{realCSn}$. If this equality is not met, the calculation process must be repeated until this condition is fulfilled. The flow rate at which this equality holds will be the actual flow rate through the secondary conductor n, and the last calculated friction factor will be the final one for the pipeline. The relative error stopping criterion for this iterative process can be taken as 0.1%, as a reference value.

Step 4. Calculate the flow rates through the secondary discharge pipes, Q_{TISn}.

Given the dynamic levels in the pumping station wells and the piezometric heads at the secondary nodes, NS_n, and using equation (8), we proceed to determine the flow rates through each of the pipes:

$$Z_{Sn} + H_{Bn} = CP_{NSn} + hf_{TISn} = Z_{Sn} + A_n + B_nQ_{TISn} + C_nQ^2_{TISn} = CP_{NSn} + K_{TISn}Q^2_{TISn} \qquad (8)$$

Where: Z_{Sn}: dynamic level at pumping station n, (m); H_{Bn}: head delivered by pump n, (m); CP_{NSn}: piezometric head at secondary node n, (m); hf_{TISn}: head losses in secondary discharge pipe n, (m); A_n: coefficient of the polynomial representing the H vs. Q curve that defines the value of the head developed by pump n for zero flow, (m); B_n, C_n: coefficients of the polynomial representing the H vs. Q curve that are a function of the head losses inside pump n, (s/m^2), (s^2/m^5) respectively, and Q_{TISn}: flow rate pumped by pump n that circulates through the secondary discharge pipe n, (m^3/s).

The solution of equation (8) can be performed by an iterative process or directly by the Cardano-Vieta solution:

$$Q_{TISn} = \frac{B_n \pm \sqrt{B_n^2 - 4(K_{TISn} - C_n)(CP_{NSn} - A_n - Z_{sn})}}{2(K_{TISn} - C_n)} \qquad (9)$$

To do this, the friction coefficient values for the secondary discharge pipes, TIS_n and $f_{supTISn}$, must first be assumed, and the procedure outlined in the previous step must be followed. Once this iterative calculation process is complete, the flow rates and friction coefficients for each pipe will be obtained.

Step 5. Perform the flow balance at the secondary nodes NS_n.

For this, expression (10) will be used, verifying that this sum is equal to the $Q_{CS}n$ calculated in step 3:

$$Q_{CSn} = \sum_{i=1}^{n} Q_{TISi} \qquad (10)$$

If the above condition is not met, assume a new piezometric head value at node N_{Sn} and repeat the process until subsystem *n* is balanced.

Step 6. Calculate the flow rate through the main or principal pipeline, *CM, Q_{CM}*.

Using the piezometric head defined at the main node, *NP*, and the piezometric head at the regulating or compensation tank, Bernoulli's equation is applied between *NP* and the discharge tank using the following equation:

$$CP_{NP} = Z_T + hf_{CM} = Z_T + K_{CM}Q_{CM}^2 = Z_T + 8.26 \cdot 10^{-2} f_{supCM} \frac{L_{CM}}{D_{CM}^5} Q_{CM}^2 \rightarrow Q = \sqrt{\frac{CP_{NP} - Z_T}{8.26 \cdot 10^{-2} f_{supCM} \frac{L_{CM}}{D_{CM}^5}}} \qquad (11)$$

Where: Z_T: dynamic water level in the discharge tank, (m); h_{fCM}: head losses in the main conductor, (m); K_{CM}: characteristic coefficient of the main conductor, (s^2/m^5); f_{supCM}: assumed friction coefficient of the main conductor, (dimm.); L_{CM}: length of the main conductor, (m); D_{CM}: diameter of the main conductor, (m), and Q_{CM}: flow rate circulating through the main conductor, (m^3/s).

First, a friction coefficient value for the main or principal conductor CM, f_{supCM}, is assumed, and once the flow rate of the main or total conductor of the general system, Q_{CM}, is obtained, steps 3 and 4 are followed identically, now for the case of the main pipe.

Step 7. Perform the flow balance at the main node *NP*

Using expression (12), and taking into account the flows contributed by all the secondary conductors to this node, it must be verified that this sum is equal to the Q_{CM} calculated in the previous step:

$$Q_{CM} = \sum_{i=1}^{n} Q_{CSi} \qquad (12)$$

If this equality is not met, the piezometric head at the main node must be adjusted until the verification is valid. Once this is achieved, the overall hydraulic system balance must be reviewed using the system's general equilibrium equation.

$$Q_{CM} = \sum_{i=1}^{n} Q_{CSi} = \sum_{i=1}^{n} Q_{TISi} \qquad (13)$$

To better understand the described methodology, the proposed procedure for the hydraulic simulation of pumped water supply systems will be applied to a real-world example. The case study will be an interconnected pumped water supply system of pumping stations, consisting of a well field, in which four pumping stations operate under a constant and fixed demand operating scheme.

4 Application of the node iteration method as a hydraulic simulation procedure for pumped source systems

The pumped water supply system will provide continuous service 24 hours a day at a constant flow rate. Table 1 provides information on the hydraulic characteristics of the pumped water supply system, which consists of a well field (multinodal system) as illustrated in Figure 1. The piezometric head of the water level in the regulating tank, Z_t = 50.00 m (pipe discharge head in the tank). High-density polyethylene (HDPE) pipes will be used. The absolute roughness of the

HDPE material is 2.5×10^{-6} m. The nominal design pressure of these pipes will be 10 atm (PN 10). A recommended velocity range of 1–1.8 m/s (INRH 2006) has been used as an indicative value for the design and operation of the system. A kinematic viscosity value of 10^{-6} m²/s is assumed.

Table 1. Information on the hydraulic characteristics of the well field (See figure 1)

Flow Rate (L/s)		Pipe Length (m)			Elevations and Dynamic Levels (m)	
Design flow rate of the source system	250.00	Main pipeline CM / Primary pipeline CP		5000.00	Main node (NP) or central node (NC), Z_{NP}	30.00
Pump 1 design flow rate	60.00	Secondary pipeline 1 CS_1		3000.00	Secondary node 1 NS_1, Z_{NS1}	23.00
Pump 2 design flow rate	60.00	Secondary pipeline 2 CS_2		2000.00	Secondary node 2 NS_2, Z_{NS2}	25.00
Secondary pipeline 1 CS_1 design flow rate	120.00	Secondary 1 discharge pipe TIS_1		1000.00	At well 1, Z_{S1}	10,00
Pump 3 design flow rate	70.00	Secondary 2 discharge pipe TIS_2		900.00	At well 2, Z_{S2}	12.00
Pump 4 design flow rate	60.00	Secondary 3 discharge pipe TIS_3		780.00	At well 3, Z_{S3}	7.00
Design flow rate of secondary conductor 2 CS_2	130.00	Secondary 4 discharge pipe TIS_4		500.00	At well 4, Z_{S4}	8.00
Diameters of the hydraulic system pipes (m)						
D_{CM}	D_{CS1}	D_{CS2}	D_{TIS1}	D_{TIS2}	D_{TIS3}	D_{TIS4}
0.5552	0.3524	0.3966	0.2776	0.2776	0.2776	0.2776

The representative polynomials of the load-capacity characteristic curve are presented, fitted to a second-degree equation as proposed by expression (1), for the four rotodynamic pumps, with pumps 1 and 2 being equal (Q in m³/s and H in m).

$$\text{Pumps 1 and 2: } H_{B1} = 85.989 - 146.46Q - 4{,}087.63Q^2 \quad (14)$$

$$\text{Pump 3: } H_{B3} = 88.304 - 86.643Q - 4{,}692.857Q^2 \quad (15)$$

$$\text{Pump 4: } H_{B4} = 73.760 - 22.276Q - 4{,}782.313Q^2 \quad (16)$$

Step 1. Assume piezometric head values at the main node, CP_{NP}, and at the secondary nodes that feed into it, CP_{NSn}.

The calculations begin by taking initial piezometric head values at NP, NS_1, and NS_2 of 59.531 m, 75.908 m, and 64.323 m, respectively.

Step 2. Calculate the head losses in the secondary conduits, $hfCSn$.

Using equation (4), we obtain hf_{CS1}values of 16.377 m and hf_{CS2} values of 4.792 m.

Step 3. Determine the flow rates through the secondary pipes, Q_{CSn}.

Initial friction coefficients for the two secondary pipes, $f_{sup}CS1$ and $f_{sup}CS2$, are assumed to be 0.02. Based on these initial friction coefficients, the flow rates in both secondary pipes are calculated using equation (5): Q_{CS1} = 0.134013 m³/s = 134.013 L/s and Q_{CS2} = 0.119290 m³/s = 119.290 L/s. As previously mentioned, this is an iterative procedure using equations (6) and (7), with a relative error stopping criterion of 0.1%. After five iterative cycles, the following results are obtained: N_{RCS1} = 605675.99; N_{RCS2} = 468920.64; $f_{realCS1}$ = 0.012781833; $f_{realCS2}$ = 0.013340008; Q_{CS1} = 0.167636 m3/s = 167.636 L/s and Q_{CS2} = 0.146063 m³/s = 146.063 L/s.

Step 4. Calculate the flow rates through the secondary discharge pipes, Q_{TISn}.

Using the dynamic water levels in the pumping station wells and the piezometric heads at the secondary nodes, NS_n, the flow rates through the secondary discharge pipes, Q_{TISn}, are calculated. As in the previous step, the calculation of the four flow rates begins with an assumed friction coefficient of 0.02 for all four pipes, taking into account the relative error stopping criterion of 0.1%.

Using equation (8) and the equations of the representative polynomials of the characteristic H vs. Q curve of the pumps operating in the well field (14), (15) and (16) and applying the Cardano-Vieta theorem (equation 9), the circulation flow rates through the secondary discharge pipes are calculated, obtaining: Q_{TIS1} = 0.051005 m^3/s = 51.005 L/s; Q_{TIS2} = 0.054426 m^3/s = 54.426 L/s; Q_{TIS3} = 0.068990 m^3/s = 68.990 L/s and Q_{TIS4} = 0.056036 m^3/s = 56.036 L/s. In the process, the following were also calculated: N_{RTIS1} = 233,941.68; N_{RTIS2} = 249,632.56; N_{RTIS3} = 316,429.42; N_{RTIS4} = 257,017.11; $f_{realTIS1}$= 0.015159264; $f_{realTIS2}$ = 0.014976931; $f_{realTIS3}$ = 0.014340784 and $f_{realTIS4}$ = 0.014896219.

Step 5. Perform the flow balance at the secondary nodes NS_n.

By performing the flow balance at each node using expression (10), it is verified that this sum is equal to the Q_{CSn} calculated in step 3.

$$Q_{CS1} = \sum_{i=2}^{2} Q_{TISn} = Q_{TIS1} + Q_{TIS2} = 0.051005 + 0.054426 = 0.105432 \ m^3/s = 105.432 \ L/s$$

$$Q_{CS2} = \sum_{i=2}^{2} Q_{TISn} = Q_{TIS3} + Q_{TIS4} = 0{,}068990 + 0{,}056036 = 0{,}125027 \ m^3/s = 125{,}027 \ L/s$$

As can be seen, the flow rates supplied by the secondary delivery pipes to their respective secondary nodes differ from those obtained for the secondary pipelines in step 3. Since the balance condition is not met at both nodes, a new piezometric head value is assumed at both secondary nodes, and the process is repeated until both subsystems are balanced. Because both sums are less than the flow rates calculated in step 3, the piezometric heads at both secondary nodes are reduced; and vice versa if the sums are higher.

After an iterative process of five cycles for both nodes, the piezometric heads that achieve zero balance are obtained. The values of both heads are: CP_{NS1} = 69.125 m and CP_{NS2} = 63.292 m. The flow costs for both subsystems are: Q_{TIS1} = 0.060980 m^3/s = 60.980 L/s; Q_{TIS2} = 0.064084 m^3/s = 64.084 L/s; Q_{CS1} = 0.125138 m^3/s = 125.138 L/s; Q_{TIS3} = 0.070261 m^3/s = 70.261 L/s; Q_{TIS4} = 0.057741 m^3/s = 57.741 L/s and Q_{CS2} = 0.127907 m^3/s = 127.907 L/s.

Step 6. Calculate the flow rate through the main or principal pipeline, *CM,* Q_{CM}.

With the piezometric head defined at the main node in the first step, and the piezometric head at the regulating or compensation tank T, a value for the friction coefficient of the main or principal pipeline, f_{supCM}, must be assumed. With subsystems 1 and 2 balanced, and the piezometric head fixed at the main node, the calculation of the main node-discharge tank subsystem is performed. As in the steps where the flow rates are calculated, a preliminary value for the friction coefficient of the main pipeline, f_{supCM} = 0.02, is assumed. Working with equation (11), a value of Q_{CM} = 0.313712 m^3/s = 313.712 L/s is obtained, with the corresponding hydraulic variables of the system: N_{RCM} = 719,436.30; f_{realCM} = 0.012370075.

Step 7. Perform the flow balance at the main node *NP*

Through expression (12), and taking into account the flows contributed by all the secondary conductors to this node, it can be verified that the sum of these is greater than the Q_{CM} calculated in the previous step.

$$Q_{CM} = \sum_{i=1}^{2} Q_{CSi} = 0.125138 + 0.127907 = 0.253045 \ m^3/s = 253.045 \ L/s$$

In this case, since the inflow to the node is less than the outflow, the piezometric head at the main node is reduced, and vice versa. As a result of an iterative process, the piezometric head at the main node that guarantees the node's balance is determined to be: CP_{NP} = 58.116 m, with the flow rate circulating through the subsystem being Q_{CM} = 0.287363 m^3/s = 287.363 L/s.

Finally, the overall balance of the general hydraulic system is verified using equation (13):

$$Q_{CM} = \sum_{i=1}^{n} Q_{CSi} = 0.287363\ m^3/s = 287.363\ L/s \neq \sum_{i=1}^{n} Q_{TISi} = 0.253065\ m^3/s = 253.065\ L/s$$

Since the overall hydraulic system was not in equilibrium, the process was repeated from the first step, decreasing the piezometric head values at all nodes of the system, as the total flow rate contributed by the four pumps exceeded the flow rate entering the main node. For the sake of brevity, the final result is presented with the solution that achieves equilibrium in the overall hydraulic system.

Table 2. Hydraulic information of the general solution of the hydraulic system

Pipe	Q (L/s)	V (m/s)	N_R (adim.)	f (adim.)	piezometric heads (m)
TIS_1	63.512	1.049	29,1305.12	0.014557548	CP_{NS1} (m) 67.256
TIS_2	66.549	1.099	305,235.24	0.014434487	
TIS_3	73.010	1.206	334,866.56	0.014195504	CP_{NS2} (m) 61.006
TIS_4	61.359	1.014	281,429.38	0.014649533	
CS_1	130.064	1.333	469,928.34	0.013347186	CP_{NP} (m) 56.961
$CS2$	133.116	1.077	427,354.46	0.01355846	
$CM\ o\ CP$	264.242	1.091	605,985.55	0.012734024	$CP_t = Z_t = 50.00$ m

Solving this example, which is considered simple, required a considerable amount of time, which increased with the addition of new subsystems. To expedite the calculations, we propose using Excel's Solver tool, which allows us to obtain an optimal solution for various decision problems by considering a performance measure (objective function), parameters, decision variables, and constraints. This way, we can adjust the values in the decision variable cells (piezometric heads at the nodes) to comply with the constraint cell limits (range of solution values for the piezometric heads at the nodes) and yield the desired result in the objective cell (balance equation at the node). Using this tool, we obtained the results presented in Table 2.

In this regard, two equations are presented that provide support for calculating the limits of the range of piezometric head values at the nodes. These expressions have been formulated for domestically manufactured HDPE pipes of class PE-100, operating with a practical velocity range of 0.5 m/s to 2.0 m/s and an internal diameter range of 129.6 mm (PN 16 atm.) to 937.8 mm (PN 5 atm.). With both equations, the slopes of the minimum and maximum energy gradients can be calculated, which will be used to determine the minimum and maximum values that the piezometric head can reach at the nodes.

$$S_{hfmín.} = \frac{hf_{mín}}{L} = 1.661828 \cdot 10^{-4} D^{-1.197798} \qquad (17)$$

$$S_{hfmáx.} = \frac{hf_{máx.}}{L} = 2.121091 \cdot 10^{-3} D^{-1.175441} \qquad (18)$$

Where: $S_{hfmin.} = hf_{min.}/L$: slope of the minimum energy grade or minimum unit head loss for a conduit, (dimensionless); $S_{hfmax.} = hf_{max.}$/L: slope of the maximum energy grade or maximum unit head loss for a conduit, (dimensionless), and D: diameter of the conduit, (m).

Both extreme values at a particular node will be determined from the piezometric head of the preceding (downstream) node. For this case study, the limit values of the piezometric head range would be determined as follows:

$$CP_{NPmín.} = CP_T + S_{hfmín.} L_{CM};\ CP_{NS1,2mín.} = CP_{NPmín.} + S_{hfmín.} L_{CS1,2} \qquad (19)$$

$$CP_{NPmáx.} = CP_T + S_{hfmáx.}. L_{CM};\ CP_{NS1,2máx.} = CP_{NPmáx.} + S_{hfmáx.}. L_{CS1,2} \qquad (20)$$

5 Conclusion

Hydraulic simulation of pumped water supply systems is important for obtaining answers about the operation of these systems, which will be taken into account in their management. As mentioned, no solution has been found in the specialized literature for this type of branched system, which has a unique characteristic (unification of branches and flow rates instead of bifurcation and division), despite being a very common hydraulic system in water supply works for the population.

One of the iterative solution methods commonly used in the hydraulic analysis of branched source systems is the Three Tank Problem. This iterative solution method involves assuming piezometric head values at the nodes based on the flow balance values, ΔQ (mass balance equation), increasing these values if ΔQ is positive, and vice versa, for each subsystem that makes up the overall hydraulic system and for the system as a whole. This is the conceptual basis of the Nodal Iteration Method developed in this work.

This article, the first in a series on this topic, focuses on formulating the methodology of the Nodal Iteration Method. This procedure is based on an analysis approach using component subsystems of the overall hydraulic system, offering a solution procedure to find the piezometric head values at the nodes that result in the overall hydraulic solution of the system.

Conflicts of interest

The author declares no conflicts of interest regarding the publication of this paper.

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Authors' Contributions

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Participated in developing the calculation methodology for applying the Iteration Method at the node, as well as in processing the data for the calculation example, contributing to its analysis and interpretation. She also participated in the search for bibliographic references and in writing the paper.

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Contributed to the research design and the analysis of the case study results, as well as to the search for bibliographic information, review, and writing of the final version of the paper.