Original Article

A Fractional PID Controller Based on Particle Swarm Optimization Algorithm

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ABSTRACT

Fractional PID controller is a convenient fractional structure that has been used to solve many problems in automatic control. The fractional scale proportional-integral-differential controller is a generalization of the integer order PID controller in the complex domain. By introducing two adjustable parameters μ and λ, the controller parameter tuning range becomes larger, but the parameter design becomes more complex. This paper presents a new method for the design of fractional PID controllers. Specifically, the parameters of a fractional PID controller are optimized by a particle swarm optimization algorithm. Our simulation results on cold rolling APC system show that the designed controller can achieve control accuracy higher than that of a traditional PID controller.

Keywords: Fractional PID controller; Particle swarm optimization; Parameter tuning

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1. Introduction

With the continuous development of science and technology, computer technology has entered a new stage of development. The actual control system has higher and higher accuracy requirements. The control field attaches great importance to fractional order system, and achieves a breakthrough in controller design. Lead corrector and lag compensator have been applied to fractional order control system in practical engineering. Networked control system has the characteristics of time delay and packet loss. Differential and integral orders are not integers in fractional order controllers. Compared with traditional proportional-integral-differential controllers, two variable parameters are added, and the structure becomes more complex in the process of improving the accuracy of parameters[1]. Therefore, the current emphasis is on the study of parameter tuning methods for fractional order control systems.

Fractional calculus is more complex than integer calculus, and common systems can be identified by integer calculus. The design and implementation of fractional calculus controller is the core of the current research of fractional calculus. Fractional order controller is introduced to fractional order controller based on integer order controller. Two parameters, integral order and differential order, are added to improve the flexibility of control form. Compared with an integer order PID controller, better dynamic performance can also be achieved by a fractional PID controller[2].

Compared with integer-order calculus, fractional-order calculus is more suitable for analyzing objects with fractional-order properties. There are many requirements for the use of integer-order calculus, such as that the factors of calculus are all real or complex integers. These requirements are relatively low in the process of fractional calculus. Any real or complex order can be

selected as its calculus factor, and fractional calculus can be regarded as an extension of integer order to any order. In the eighteenth century, there was a study on fractional order, which began to be applied in the field of control in the middle of the last century. Fractional order control system can meet the requirements of integer order system control in the field of control. For fractional order system model, fractional order control system can achieve good results. Therefore, it is widely used in biological analysis and other related fields, and the system specific characteristics of integer-order calculus are more powerful. With the continuous development of science and technology, fractional calculus has been used in the development and research of many disciplines. The corresponding models of fractional calculus have been widely used in the control system theory and economics research fields, and attracted worldwide attention.

The key to the design and parameter tuning of fractional order controllers is to study fractional order. There are many theoretical results about parameter tuning and control system research, some of which have been achieved. The main research contents are the application of correlation optimization algorithm in controller design and the stability analysis of fractional order controller system.

In [14], the numerical solution of fractional order control system is studied, and a new design idea of fractional order controller is demonstrated. The design of fractional order system controller with rational order is also proposed. The solution of controller control parameters is discussed in [15] and [16]. In [3] and [4], the solution of phase margin and amplitude margin are introduced in the process of parameter solution. In [17], the construction method of fractional-order-delayed-PD controller is studied, and the process of controller construction for a single type of second-order time-delay system is analyzed in detail. The effect of integer-order-controller based on Z-N method and the control effect of a class of second-order-delay-uPD controller are compared and analyzed. The control result of fractional-order-uPD controller is better than that of integer-order-controller. Conditions are demonstrated. In [18], the parameter tuning of fractional-order-controller is realized by visual images, the amplitude tuning process

of fractional-order controller is analyzed, and the direction of time-delay optimization and design ideas of fractional-order controller are explored. The parameter tuning of fractional-order controller is studied. The fractional-order controllers with different working states are created. The working states are divided into two categories: control and parameter tuning. The purpose of analyzing fractional-order PI controller is to realize the self-tuning of control parameters. The related theory of intelligent control is also analyzed. The actual verification process is also completed by means of digital implementation.

In order to optimize the design process algorithm of fractional order controller, the optimization method used in the traditional controller parameter tuning process is applied in the design parameter tuning of fractional order controller. The commonly used methods are neural network, genetic algorithm and so on[19]. The improved particle swarm optimization algorithm is used to process the parameter setting process, so that the set particle coordinates are guaranteed in the search space. The optimization of controller performance is completed, and the validity of the above design process is demonstrated by continuous fractional expansion simulation[20]. The hardware of the design and analysis controller is programmable gate array[21]. The parameters of fractional order controller are tuned and analyzed by means of neural network to achieve the effect of parameter self-tuning. The genetic algorithm is used to simulate the analysis of parameter tuning, and the continuous fractional expansion and operator are applied to the simulation process.

The fractional order controller is designed and studied, and the design process of the controller is completed by combining the control flow and strategy[22]. The fractional-order controller is designed based on the existing inner-membrane control theory. In the process of design and research, the fractional-order correlation model reduction operation is processed by stochastic search optimization, and the related parameters of the fractional-order controller are tuned. The internal-model control correlation principle is used. In [23], the adaptive control process of fractional order controller model is applied in boiler system. The analysis of boiler load is completed by simulation. Through simulation analysis,

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the adaptive control of boiler is realized by fractional order controller. Without changing the parameters set by the controller, the boiler temperature is effectively controlled, so that the anti-interference ability and control stability can be achieved. In [24], the generalized predictive model of fractional order controller is analyzed and discussed. The fractional operator of the cost equation of the controller is realized in the process. A more rigorous mathematical derivation of the generalized predictive model of fractional order controllers is given, and the predictive results of the generalized control of integer order controllers are compared and analyzed. The advantages of the generalized predictive model of fractional order controllers are highlighted. In the design of fractional order controller, the internal model controller is extended and analyzed. Expected bandwidth and two degrees of freedom are introduced. The defect that the original fractional order controller cannot track the given value in the control process is remedied. The supplementary process is completed by mathematical derivation.

In this paper, we develop a new design method for the tuning of the parameters of fractional PID controllers. We utilize the fact that particle swarm optimization (PSO) algorithms have excellent ability to search for the global optima of multi-dimensional functions and develop a PSO based approach to optimize the control accuracy of a fractional PID controller. Our simulation results on a cold rolling APC system show that the designed controller can achieve control accuracy higher than that of a traditional fractional PID controller.

2. The Proposed Approach

2.1 Fractional PID control algorithm

Integer order calculus theory can be generalized, which can be expressed as simultaneous fractional order calculus. Integer is no longer the only order of calculus. Any real number or even complex number can become the order of calculus. It is widely used in many fields such as biomedicine. Combining fractional calculus with a traditional PID controller can lead to a fractional order PID controller, which improves dynamic performance and robustness, and provides more flexible controller design to ensure better control of cold rolling system under complex conditions.

Fractional calculus operators are defined as follows.

$$
{}_aD_t^\alpha = \begin{cases} \dfrac{d^\alpha}{dt^\alpha} & \mathrm{Re}(\alpha) > 0 \\ 1 & \mathrm{Re}(\alpha) = 0 \\ \displaystyle\int_a^t (dt)^{-\alpha} & \mathrm{Re}(\alpha) < 0 \end{cases}
$$

(1)

The upper and lower bounds of operator operations are a and t, the order of Operator Calculus is the real part represented by = 0, the differential part represented by > 0 and the integral part represented by < 0. There is no unified standard definition of fractional calculus. In current research, there are generally three definitions, including *Grunwald - Letnikov*, *Caputo* and *Riemann - Liouville*.

In the process of numerical calculation, The *Grunwald - Letnikov* definition is generally adopted. Based on the expression of higher order derivatives, the order of calculus is extended from integer order to fractional order from the classical definition of calculus. Its formula is described as follows.

$$
{}_aD_t^\alpha f(x) = \lim_{h \to 0} h^{-\alpha} \sum_{i=0}^{(x-a)/h} (-1)^i \binom{\alpha}{i} f(x - ih)
$$

(2)

Riemann - Liouville definition is also widely used in Calculus of fractional order. First, the integral is defined by RL, and then the definition of related differential is deduced through the definition of fractional order integral. Its formula is described as follows:

$$
{}_aD_t^\alpha f(x) = \frac{d^m}{dt^m} \left[\frac{1}{\Gamma(\alpha)} \int_a^x \frac{f(\tau)}{(x-t)^{\alpha-m+1}} d\tau \right]
$$

(3)

The corresponding integral definitions are expressed as follows.

$$
{}_aI_t^\alpha f(x) = \frac{1}{\Gamma(\alpha)} \int_a^x (x-\tau)^{\alpha-1} f(\tau) d\tau
$$

(4)

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The α order differential of a function $f(\tau)$ defined by *Caputo* fractional calculus can be expressed as follows.

$$\begin{cases} {}_aD_t^\alpha x(t) = Ax(t) + Bu(t) \\ \quad y(t) = Cx(t) + Du(t) \end{cases}$$

(5)

$\alpha = m + \gamma$, $m \in N$, $0 < \gamma < 1$, *Caputo* definition defines the α order integral of a function $f(\tau)$ is as follows.

$$_aI_t^\alpha = \frac{1}{\Gamma(m-\alpha)} \int_a^t \frac{f^m(\tau)}{(t-\tau)^{\alpha-m+1}} d\tau$$

(6)

Unify the differential and integral expressions, *Caputo* fractional calculus is defined as.

$$_aD_t^\alpha = \frac{1}{\Gamma(m-\alpha)} \int_a^t \frac{f^m(\tau)}{(t-\tau)^{\alpha-m+1}} d\tau$$

(7)

Through the analysis of the above three definitions of fractional calculus, it is found that both *Riemann - Liouville* and *Caputo* definitions are improved from the *Grunwald - Letnikov* definition. The *Riemann - Liouville* definition can simplify the calculation of fractional derivatives, and for *Caputo* definition, the Lagrangian transformation is more advantageous to the solution of fractional calculus equation because of its simpler definition.

In the process of numerical simulation of fractional order control system, the fractional order transfer function is only used directly, so the direct simulation cannot be realized. The fractional order operator can be approximated and discretized first, then the fractional order controller system can be simulated. At present, the most often used approximation method is the direct approximation method.

The direct approximation method is used to approximate the fractional-order system by Z-transform. The original fractional-order system is transformed into a discrete-state integer-order system after Z-transform. Then the integer-order system is simulated numerically, which is the specific process. The processing methods used include: direct power series expansion of Euler operator, continued fraction expansion of the original fractional order system through the use of operator to achieve. After the indirect approximation of fractional-order control system is completed, the original fractional-order controller system is transformed into a continuous integer-order controller system through transformation, and the approximation of fractional-order control system is successfully completed. That is to say, the transfer function is transformed into integer order state by approximation, and then the integer order controller system is further processed. The system controlled by fractional-order controllers is similar to the system controlled by integer-order controllers. There are many different expression models, including the description in the form of transfer function, the description in the form of fractional-order differential equations, and the description in the state space. The form of function realizes the description of the controller system.

Power series are used to approximate a discrete Euler operator. That is $w(x^{-1}) = (1 - x^{-1})^\alpha$. Then the power series expansion of $(1 - x^{-1})^\alpha$ is carried out, which can be obtained by relevant definitions as follows.

$$\nabla_T^\alpha = T^{-\alpha} \sum_{k=0}^{\infty} (-1)^k \binom{\alpha}{k} f((n-k)T)$$

(8)

By expanding $(1 - z^{-1})^\alpha$ into exponential series, the following forms can be obtained.

$$\nabla_T^{-\alpha} = T^\alpha \sum_{k=0}^{\infty} (-1)^k \binom{-\alpha}{k} f((n-k)T)$$

(9)

The results of approximation of differential operators for fractional order systems are as follows.

$$Y(x) = T^\pm PSE\left\{(1 - X^{-1})^{\pm\alpha}\right\} F(x)$$

(10)

In the above expression, the sampling period of the differential operator is T, the system approximates the Z-transform result of the output sequence is described by $Y(x)$, the Z-transform result of the output sequence $f(nT)$ is expressed by $F(x)$, and the power series expansion of function u is expressed by *PSE*.

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The differential equation of fractional order controller is described by the following equation.

$$u(t) = k_p e(t) + k_i D_T^{-\lambda} e(t) + k_D D_t^{\delta} e(t)$$

(11)

The continuous transfer function of FOPID is obtained by Laplace transformation and is given by the following equation.

The design of FOPID controller involves three parameters and two first-order parameters, which are not necessarily integers. The fractional order controller generalizes the traditional integer order controller. This extension can provide greater flexibility for achieving control objectives.

2.2 Particle swarm optimization algorithm

For particle swarm optimization, it is a simulated particle swarm optimization algorithm based on population stochastic optimization algorithm and social behavior of birds. It is inspired by the observation of the feeding behaviors of birds. The swarm intelligence model comes from the simulation of birds' swarm behavior. The optimal solution is found through coordination and information sharing among individuals in the swarm: assume a random food pile. In a certain space, a group of birds in random search, initially do not know the specific food location, just rely on their intuition, fly to different places to find this pile of food, when one of the birds found the food location, will provide some information to the surrounding birds, through this information exchange, after a period of time, all birds can. Finding the location of this pile of food is equivalent to focusing all the particles on one point, which is the optimal solution we need.

For the analysis of particle swarm optimization, the search space randomly disperses all particles, and the optimal solution in the search space may be each particle. For particles, the direction of movement is determined by the intersection of flight velocities. After the first movement, each particle will calculate the fitness of all particles through the fitness function. Individuals constantly update the fitness of all particles. Comparing the fitness value with the group optimum value in the particle swarm, the condition of replacing the group optimum value is that the result is better. At present, the individual optimum value and the group optimum value are used to correct the direction and velocity of the next particle. After many iterations, the particles will be concentrated near the optimal solution, which is the particle with the best fitness.

The core operation is the particle update formula, i.e. the velocity and position update formula. The global optimization model is as follows.

$$v_i(k+1) = w v_i(k) + c_1 r_1 (p_1(k) - x_1(k)) + c_2 r_2 (p_g(k) - x_g(k))$$

(12)

$$x_i(k+1) = x_i(k) + v_i(k+1)$$

(13)

Among them, w is the inertia weight coefficient, $v_i(k)$ and $x_i(k)$ are the velocity and position of the particle i in generation k respectively, $p_1(k)$ and $p_g(k)$ are the optimal position and the global optimal position of particle i and all particles, the acceleration coefficients are c_1 and c_2, r_1 and r_2 are uniformly distributed in [0,1]. The empirical part is the first part, which provides a necessary momentum to the particle to make its inertial motion accord with the diversity of the particle according to the current self-motion and the degree of trust of the particle; the memory of the best cognitive position is the second part, which encourages the particle to fly to the best position found by itself, and corresponds to the characteristics of centralization of the search process; the social cognitive part is the third part. It represents information sharing and cooperation, makes particles fly to the optimal position in particle swarm optimization, and corresponds to the characteristics of mutual restriction and balance between populations.

3. Simulation Results

An example of a cold rolling APC system transfer function is as follows.

$$G(s) = \frac{0.33}{3.601 \times 10^{-6} S^3 + 0.002657 S^2 + S}$$

(14)

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For the whole cold rolling fractional order PID system, after the system design is basically completed, the whole system is tested, and some minor errors in the design process are found and corrected. The knowledge used in the previous design is applied to the specific testing process, so the testing process is difficult but crucial. First, we will give a brief introduction to the system simulation environment MATLAB. Then I will compare the two systems by simulation. One is the simulation of APC system with fractional order PID, the other is the simulation of APC system with fractional order PID based on particle swarm optimization. Then the two test results are compared and analyzed, and the conclusion is drawn.

The objective that needs to be minimized by PSO is as follows.

$$IAE = \int_0^\infty |r(t) - y(t)| dt = \int_0^\infty |e(t)| dt$$

(15)

Before calling the particle swarm optimization algorithm, we try to use fractional order PID system to adjust the objective function of cold rolling APC system manually. **Figure 1** is the step response and Bird diagram of the objective function manually adjusted by fractional order PID controller. The overshoot of the step response of the system is not more than 10%, but it has not reached the optimal level. If we only adjust it manually, we can make it possible. If the system becomes better, the work will become enormous. If we can use particle swarm optimization to tune parameters, it will greatly reduce our workload. The advantages of particle swarm optimization can be embodied vividly, and the optimal parameters can be obtained by calling the particle swarm optimization algorithm. The step response of APC fractional-order PID system after particle swarm optimization is shown in **Figure 2**. We can see the difference more intuitively by putting the two step response diagrams in **Figure 3**. From the comparison of **Figure 3**, we can clearly see that the overshoot of APC fractional-order PID system after parameter tuning by particle swarm optimization is very small, and the adjustment time is also very large. Reduction. In this way, the optimal parameters of fractional order PID system can be obtained. **Figure 1**

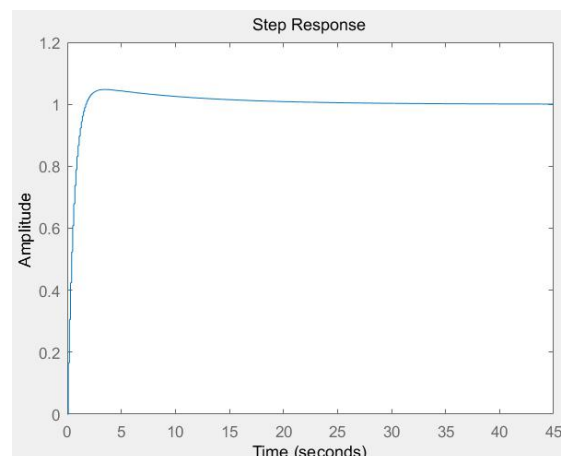and **Figure 2** fractional order PID controller parameters are listed in **Table 1**.

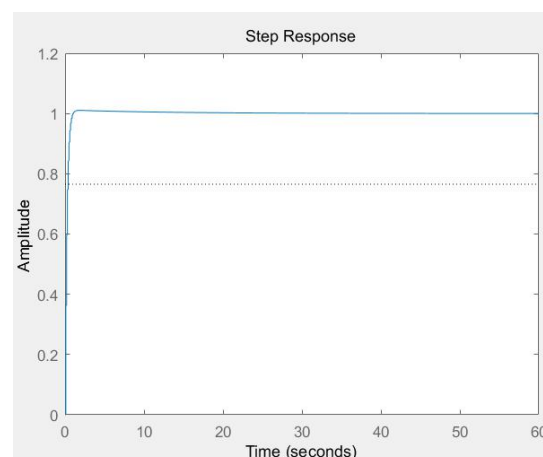**Figure 1.** Step response diagram of manual parametric adjustment.

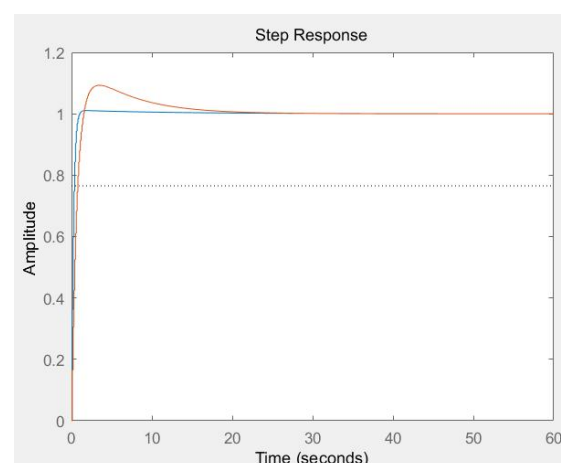**Figure 2.** Step response diagram after particle swarm optimization is used to set parameters.

Figure 3. A diagram that compares the results shown in **Figure 1** and **Figure 2**.

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Table 1. Fractional order PID controller parameters

	k_p	k_i	k_d	μ	λ
Manual	5	0.8	0	1.2	0.6
PSO	11	0.5	0	0.8	1.6

4. Conclusions

The fractional order PID controller based on particle swarm optimization can effectively control the APC system of cold rolling. The fractional order PID system is used to replace the traditional PID system and the particle swarm optimization algorithm is selected to adjust the initial parameters of the fractional order PID controller according to the requirements of the system. The simulation results show that the fractional order PID controller based on PSO algorithm makes APC system follow the input signal quickly and accurately, and achieves effective control of the non-linear servo system, so that its anti-interference ability and anti-system deformation ability are relatively strong.

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