Force Transmission Caused by Water Flow during Collisions Using Water as a Medium

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Abstract: This study explores the mechanism of force transmission caused by water flow during collisions using water as a medium. By establishing a theoretical model and conducting systematic experiments, the study investigates the effects of factors such as water volume and release height on the rebound height of a ping-pong ball. A mathematical model incorporating fluid dynamics and air resistance was developed, and its validity was verified through CFD simulations and high-speed photography. Experimental results show that when the water volume reaches approximately 310 ml, further increases in water volume have minimal effect on rebound height, thereby confirming the "infinite water depth" assumption. A linear relationship was observed between release height and post-collision velocity, with an empirical constant of approximately 4.31. The study also observed high-speed jet phenomena similar to the "Pokrovsky experiment". This research provides new insights into the mechanism of force transmission by water as a medium during collision processes.
***Keywords*:** water medium; collision process; force transmission

1. Introduction

The energy transfer in collision processes is a classic and complex physics problem, particularly when the medium transitions from a solid to a fluid, where the mechanical mechanisms involved become even more intricate. This study focuses on collision processes using water as the medium, aiming to uncover the mechanisms of force transmission caused by water flow. This problem not only holds theoretical significance but also has important engineering applications in areas such as hydropower generation and ocean engineering.Previous studies have primarily concentrated on solid collisions and simple fluid collisions, with relatively limited research on complex fluid media. The work of Antkowiak et al. (2007) on the short-term dynamic behavior of density interfaces following impacts provides an important reference for this study. The theoretical research by Gordillo and Blanco-Rodríguez (2023) on jetting after cavitation, as well as the studies by Gordillo et al. (2020) on pulse jets produced by fluid focusing, offer a theoretical foundation for understanding the high-speed jet phenomena observed in this experiment.This study innovatively examines the system comprising a ping-pong ball, water, and a container as a whole. By combining theoretical modeling, numerical simulation, and systematic experiments, it delves into the mechanism of force transmission by water as a medium during collision processes. The research focuses on the effects of factors such as water volume and release height on the rebound height of the ping-pong ball, while capturing fluid dynamic phenomena during the collision process using high-speed photography.The goal of this study is to establish a theoretical model capable of accurately describing collision processes using water as the medium and to validate the model through experiments. The results are expected to provide theoretical guidance for related engineering applications while contributing new knowledge to the fields of fluid dynamics and collision mechanics.

2. Theoretical Analysis

2.1 Establishing the Theoretical Model and Assumptions

To study the motion characteristics of a ping-pong ball dropping into a water-filled container and colliding with the surface, a theoretical model was established. A cylindrical coordinate system (ε,ϕ,z) was set up within the container, while a spherical coordinate system (r,θ,ϕ) was established at the concave spherical cap of the ball, where the radius of the spherical cap is R and the radius of the container is R/λ. For simplification, three assumptions were made:

(1)Neglecting the effect of surface tension. The Weber number, calculated as shown in Equation (1), demonstrates this simplification:

$$\mathrm{We} = \frac{\rho v_0^2 R}{\gamma} \tag{1}$$

In the equation,We is the Weber number, used to measure the ratio of inertial forces to surface tension forces;ρ is the density of the liquid; v_0 is the initial velocity of the ping-pong ball: R is the radius of the spherical cap of the ball. γ is the surface tension coefficient of the liquid.

The Weber number is on the order of 10^4, significantly greater than 1. Therefore, the influence of surface tension on velocity can be ignored.

(2)Neglecting the effect of viscous forces. The Reynolds number, calculated as shown in Equation (2), supports this assumption:

$$\mathrm{Re} = \frac{Rv_0}{v_\tau} \tag{2}$$

In the equation,Re is the Reynolds number, used to measure the ratio of inertial forces to viscous forces; v_τ is the density of the liquid.The Reynolds number is on the order of 10^5, indicating turbulent flow. Therefore, viscous forces can be neglected.

(3)Assuming infinite water depth. When the water volume V>340ml, further increases in water depth have a negligible effect on the rebound height, allowing the water to be treated as infinitely deep.

These assumptions enable the study to establish the Poisson equation. By solving this equation, the pressure field and velocity field can be obtained, allowing for the analysis of energy transfer throughout the collision process.

2.2 Qualitative Analysis of the Motion Process

2.2.1 Pre-collision State

In the pre-collision state, the entire system—composed of the container, water, and ping-pong ball—falls with uniform acceleration under the influence of gravity. Due to the relatively large total weight of the system, the effect of air resistance during the descent can be approximately neglected. At this stage, the water surface forms a nearly spherical concave depression, which is critical for the subsequent collision process. Ultimately, under the influence of adhesive forces, the ping-pong ball is slightly pulled into the liquid surface.Although the theoretical model neglects the influence of surface tension, in practice, surface tension may have a minor impact on the initial position of the ping-pong ball within the container. However, since the calculated Weber number is significantly greater than 1, this effect can be disregarded in the overall motion. Similarly, while the study assumes infinite water depth, experimental results show that when the water volume reaches approximately 340 ml, further increases in water volume have minimal impact on the final rebound height. Therefore, this assumption is reasonable for practical applications.During this stage, the gravitational potential energy of the entire system is gradually converted into kinetic energy, preparing for the subsequent collision process.

2.2.2 Instantaneous Collision Process

The instantaneous collision process is the most critical phase of the phenomenon. At the moment when the bottom of the container contacts the surface, the uneven distribution of the pressure field causes the velocity vector of the water flow to quickly point toward the spherical cap region. This results in a high-speed jet ejection from the center, forming a phenomenon akin to the "Pokrovsky experiment."In this process, water, as the medium, transfers the energy of the system to the ping-pong ball located at the concave spherical cap, efficiently converting the initial gravitational potential energy of the system into the kinetic energy of the ping-pong ball.At this moment, the flow exhibits turbulent characteristics, with the Reynolds number reaching approximately 10^5, allowing the study to neglect the influence of viscous forces. Furthermore, due to the large forces involved during the collision process, the effect of gravity can also be neglected. A simplified form of the Navier-Stokes equation can be used to describe fluid motion.

By solving the Laplace equation, the study determines the distribution of the pressure field, which is then used to derive the velocity of the ping-pong ball post-collision.

2.2.3 Post-collision State

After the collision, the ping-pong ball separates from the system composed of the container, water, and the ball itself, entering an independent motion phase. At this point, the ball is essentially unaffected by forces in the horizontal direction. However, in the vertical direction, the ball is subjected to both gravitational force and air resistance.Since the ball's speed does not exceed 20 m/s, the study simplifies air resistance to a form proportional to the velocity, as shown in Equation (3):

$$f = kv \tag{3}$$

In the equation, f is the air resistance; k is the air resistance coefficient; v is the velocity of the ping-pong ball.

By integrating the equation of motion, the study derived an equation describing the motion of the ball, as shown in Equation (4):

$$v = \left(v(\tau) + \frac{mg}{k} \right) e^{-\frac{kt}{m}} - \frac{mg}{k} \tag{4}$$

In the equation, v is the ball's velocity in the vertical direction; $v(\tau)$ is the initial velocity after the collision; m is the mass of the ping-pong ball; g is the acceleration due to gravity; t is time.

Further integration yields the maximum height L the ball can reach, as shown in Equation (5):

$$L = \frac{mv(\tau)}{k} - \frac{gm^2}{k^2} \ln\left(\frac{kv(\tau)}{mg} + 1 \right) \tag{5}$$

In the equation, L is the maximum height achieved by the ping-pong ball after being ejected; ln is the natural logarithm.

2.3 Establishing the Mathematical Model

Based on the previous theoretical analysis and assumptions, the study can establish a mathematical model to describe the entire collision process. First, the simplified Navier-Stokes equation is used to describe the fluid motion, as shown in Equation (6):

$$\rho \frac{\partial v}{\partial t} = -\nabla p \tag{6}$$

In the equation, ρ is the density of the liquid; $\frac{\partial v}{\partial t}$ is the partial derivative of velocity with respect to time, representing the rate of change of velocity; ∇p is the pressure gradient.

After integrating the time term, the velocity after the impact is obtained, as shown in Equation (7):

$$v(\tau) = v(0) - \frac{1}{\rho} \nabla P \tag{7}$$

In the equation, $v(\tau)$ is the velocity after the collision; v(0) is the velocity before the collision; ∇P is the pressure gradient; P is the pressure pulse.

Since water is nearly incompressible, P satisfies the Laplace equation, as shown in Equation (8):

$$\nabla^2 P = 0 \tag{8}$$

In the equation, ∇^2 is the Laplace operator, representing the second-order spatial derivative of the pressure field.

By solving this equation, the study can obtain the distribution of the pressure field, as shown in Equation (9):

$$P(r,\theta) = -\frac{rP_1(\cos\theta)}{R} + \sum A_{2s} F_{2s} \tag{9}$$

In the equation, $P(r,\theta)$ is the pressure field distribution in the spherical coordinate system; r is the radial distance; θ is the polar angle; P_1 is the pressure distribution function related to the angle; R is the radius of the spherical cap, A_{2s} 、 F_{2s} are coefficients and functions related to the shape and size of the container.

Further, the study can obtain the velocity of the ping-pong ball after the collision, as shown in Equation (10):

$$v(\tau) = Av(0) \tag{10}$$

In the equation, $v(\tau)$ is the velocity after the collision; $v(0)$ is the velocity before the collision; A is the empirical constant, experimentally measured to be approximately 4.31.

For the post-collision state, considering air resistance, the equation of motion for the ping-pong ball is given by Equation (11):

$$v = \left(v(\tau) + \frac{mg}{k} \right) e^{-\frac{kt}{m}} - \frac{mg}{k} \tag{11}$$

In the equation, v is the velocity of the ping-pong ball at any given moment; $v(\tau)$ is the initial velocity after the collision; m is the mass of the ping-pong ball; g is the acceleration due to gravity; k is a constant related to air resistance; t is time.

Integrating the equation gives the maximum rebound height, as shown in Equation (12):

$$L = \frac{mv(\tau)}{k} - \frac{gm^2}{k^2} \ln\left(\frac{kv(\tau)}{mg} + 1 \right) \tag{12}$$

In the equation, L is the maximum rebound height of the ping-pong ball; ln is the natural logarithm.

This model covers the main physical characteristics of the entire process, including fluid dynamics, collision mechanics, and the effects of air resistance. It is capable of accurately predicting the rebound height of the ping-pong ball and explaining phenomena observed in experiments, such as the effects of water volume and release height on the rebound height. However, the model still contains some simplifications, such as neglecting the influence of surface tension and viscosity, which may lead to prediction deviations in some extreme cases. Future research could consider incorporating these factors to further improve the model.

2.4 Theoretical Model Simulation

To verify the accuracy and predictive ability of the theoretical model established in the study, numerical simulations were conducted. The simulations focused on two main aspects: fluid dynamics modeling and predicting the motion trajectory of the ping-pong ball.

First, the study used Computational Fluid Dynamics (CFD) software to simulate the instantaneous collision process. In the simulation, the same initial conditions were set as in the experiment, including container size, water volume, and initial falling velocity. The simulation results showed that at the moment the container's bottom contacts the plane, a high-speed jet forms at the center, which aligns with the theoretical predictions and experimental observations. Figure 1 displays the velocity field distribution obtained from the simulation, clearly showing the formation process of the jet.

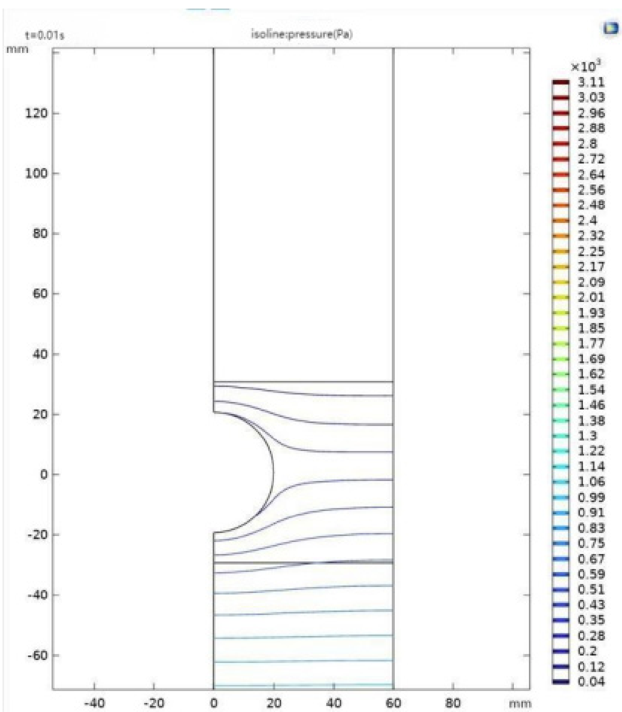

Figure 1. Simulated velocity field distribution

By comparing the simulation results with the high-speed photographic images taken in the experiment (as shown in Figure 2), the study found that there is a high degree of consistency between the two in terms of jet morphology and development process. This not only verifies the correctness of the theoretical model but also provides intuitive visual results that help in understanding the "Pokrovsky experiment" phenomenon.

Figure 2. High-speed photographic images(a.Simulated velocity field;b.Actual high-speed image)

Next, the study conducted numerical simulations of the motion trajectory of the ping-pong ball. Based on the mathematical model developed in the study, particularly considering the influence of air resistance, a program was written to predict the ball's launch height. The simulation results are shown in Figures 3 and 4, where the red curves represent the theoretical predictions, and the scatter points represent the experimental data. From the figures, it can be seen that the theoretical prediction curves match well with the experimental data points, especially in the relationships between v(0)-v(τ) and v(τ)-L.

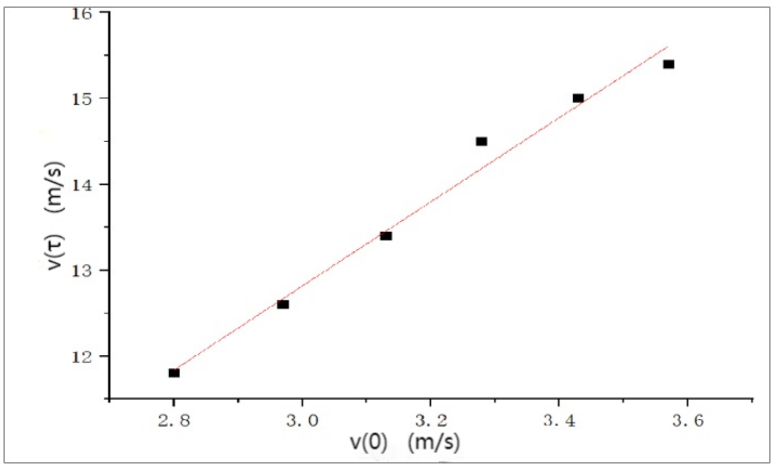

Figure 3.v(0)-v(τ)

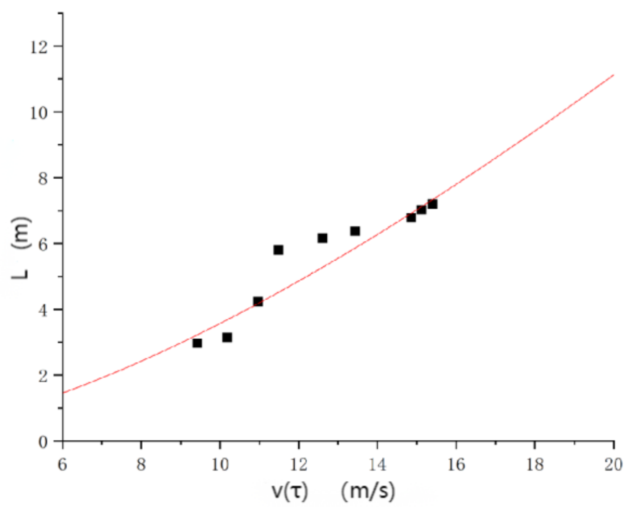

Figure 4. v(τ)-L

These simulation results not only validate the effectiveness of the theoretical model but also help the study gain a deeper understanding of the entire physical process. For example, the simulation clearly shows that after the water volume reaches a certain level (around 310ml), further increases in water volume have little effect on the launch height, which is consistent with the experimental observations and supports the reasonableness of assuming an "infinite depth" of water in the theoretical model.

However, the study also noted that under certain extreme conditions, such as very low release heights (<30cm) or very high release heights (>100cm), there is some deviation between the simulation results and experimental data. This suggests that under these conditions, the study may need to consider factors that have been neglected in the current model, such as surface tension or cavitation effects.

3. Experimental Design and Methods

3.1 Experimental Instruments and Materials

To comprehensively study the force transmission caused by water flow during collisions with water as a medium, the following experimental instruments and materials were carefully selected:

(1) Imaging Devices: A Phantom v7.3 high-speed camera was used to capture detailed changes during the collision moment at a frame rate of 1000fps. Additionally, a standard 60fps camera was used to record the overall motion trajectory of the ping-pong ball.

(2) Container: Cylindrical containers with diameters ranging from 65mm to 90mm were mainly used. To study the impact of container material on the results, high-stiffness PVC bottles were chosen as the primary experimental containers to ensure ideal experimental conditions. Additionally, 20mm x 200mm glass test tubes were prepared to observe the details of water flow.

(3) Ping-Pong Ball: Standard 40.00mm diameter plastic shell ping-pong balls were used.

(4) Release Device: A custom iron frame and fine thread were used, allowing precise control of the release height to ensure repeatability of the experiment.

(5) Auxiliary Measurement Device: A custom white backdrop, 12m in length and marked with 5cm intervals, was used to accurately measure the ping-pong ball's launch height.

(6) Water: Regular tap water was used, and the water volume was precisely controlled using a graduated cylinder, ranging from 0ml to 400ml.

(7) Other Auxiliary Tools: Including rulers, stopwatches, thermometers, etc., to record experimental environmental parameters and assist with measurements.

The selection of these instruments and materials aims to ensure the accuracy and repeatability of the experiments. The high-speed camera captures subtle changes during the collision, while the long backdrop and dual-camera setup ensure accurate measurement of the ping-pong ball's initial speed, final speed, and maximum launch height. The choice of PVC containers is based on preliminary experiments showing that the hardness and elasticity of the container material are critical to experimental outcomes. Through these carefully selected instruments and materials, the study aims to comprehensively and accurately investigate the force transmission characteristics of water as a medium during the collision process, providing reliable experimental data to validate the theoretical model.

3.2 Experimental Subjects and Variables

The main experimental subject of this study is the system composed of a container, water, and a ping-pong ball, with a focus on the motion characteristics of the ping-pong ball after the collision. Based on theoretical analysis and preliminary experimental results, the study defines the main variables, controlled variables, and secondary variables.

Main Research Variables:

(1) Release Height (H): The study range is from 40cm to 120cm. It was observed that when the height is below 40cm, the ball's bounce effect is not significant; while heights above 120cm are limited by the equipment, making it difficult to ensure a straight fall in every experiment.

(2) Water Volume (V): The study range is from 0ml to 400ml. Special attention is given to the critical point around 310ml, as experiments found that beyond this point, further increases in water volume had minimal impact on the bounce height.

(3) Pre- and Post-Collision Speeds ($v(0)$ and $v(\tau)$): These are measured using a high-speed camera and are used to verify the speed relationships in the theoretical model.

(4) Bounce Height (L): As the primary dependent variable, it is used to evaluate the force transmission effects caused by the water flow.

Controlled Variables:

(1) Container Material: PVC bottles are mainly used due to their high stiffness, ensuring ideal experimental results.

(2) Container Size: Cylindrical containers with diameters ranging from 65mm to 90mm are used.

(3) Ping-Pong Ball Specifications: Standard 40.00mm diameter ping-pong balls are used.

(4) Ambient Temperature: The experimental environment temperature is recorded and kept relatively constant to minimize the influence of temperature on the water's viscosity.

Secondary Research Variables:

(1) Container and Ping-Pong Ball Radius Ratio (λ): Although not the primary research focus, it is addressed in the theoretical model and may have a potential impact on the results.

(2) Ground Material: Although kept consistent in this experiment, it could be a direction for future research.

By systematically adjusting these variables, particularly the release height and water volume, the study comprehensively investigates the force transmission characteristics of water as a medium during the collision process. Special attention is given to the relationships between v(0)-v(τ) and v(τ)-L to validate the theoretical model developed in the study.

3.3 Experimental Procedure

The research team developed a series of carefully designed experimental steps aimed at systematically studying the effects of water volume and release height on the bounce height of the ping-pong ball, validating the predicted speed relationships in the theoretical model, and observing the detailed phenomena during the collision process. The team carried out detailed preliminary preparations, including calibrating all measurement equipment, adjusting the release device, and setting up the 12m long white curtain for accurate height measurements. In Water Volume Influence Experiment, the researchers used PVC containers and gradually added water starting from 0ml, increasing by 20ml increments until reaching 400ml. The release height was fixed at 80cm, and each group was repeated 3 times. Release Height Influence Experiment focused on the optimal water volume (estimated to be around 310ml). The release heights were tested from 40cm to 120cm, with intervals of 10cm for each group. The velocity relationship verification experiment uses a high-speed camera (1000fps) to precisely capture the instantaneous velocities before and after the collision. The velocities v(0) and v(τ) were measured in the range from 40-120cm of release height, as shown in Figure 5.

Figure 5. High-speed photography equipment and experimental setup layout

The study also included container material comparison experiments and detailed observation experiments, with the latter using a 20mm*200mm glass test tube to focus on observing the details of water flow. Throughout the experiments, high-speed cameras were used to record the collision process, and regular cameras captured the overall trajectory, while the ejection height (L) was carefully recorded for each trial. During the data collection and organization phase, the team performed statistical analysis on each set of experimental data, calculating the average and standard deviation, and created relevant charts. Throughout the entire experiment, the researchers continuously monitored and recorded the laboratory temperature, ensuring a level experimental surface to minimize external interference. These experiments not only validated the theoretical model of the study but also provided important experimental evidence for further optimization of the model. Additionally, the study observed that once the water volume reached approximately 310ml, further increases in water volume had a diminished effect on the ejection height. This observation provided significant support for validating the study's "infinite depth" hypothesis.

4. Experimental Results and Analysis

Through systematic experiments and data collection, the study obtained rich experimental results. Regarding the effect of water volume on ejection height, the experimental results are shown in Figure 6. The study found that:

(1) When the water volume is less than 310ml, the ejection height increases with the increase in water volume, presenting an approximately linear relationship.

(2) Once the water volume reaches approximately 310ml, the ejection height no longer changes significantly with the increase in water volume.

(3) This result validates the reasonableness of the "infinite depth" assumption in the theoretical model and provides a basis for selecting 340ml as the standard water volume for subsequent experiments.

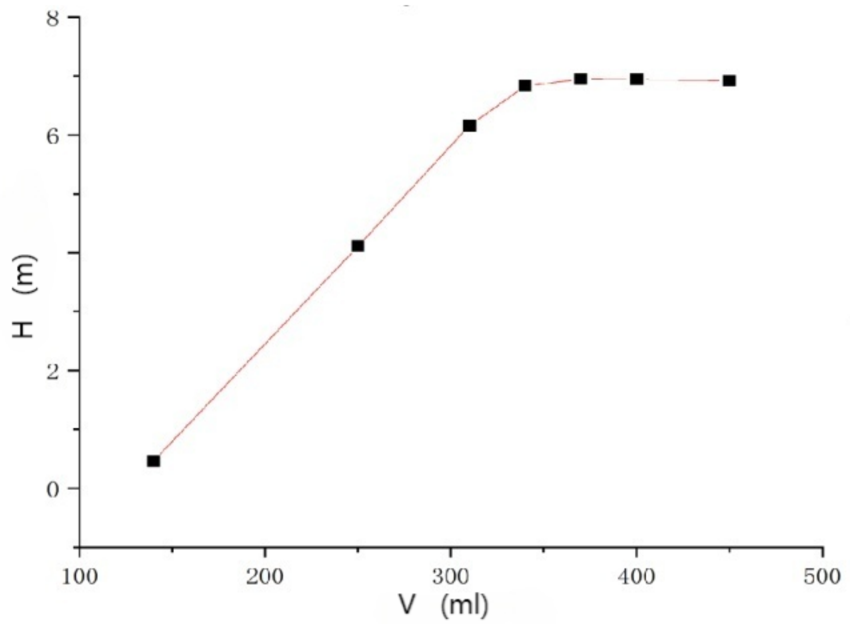

Figure 6. H-V graph

The impact of release height on ejection speed, as shown in Figure 3 above:

(1) In the release height range of 40-80 cm, the relationship between $v(\tau)$ and $v(0)$ shows a good linear correlation.

(2) The fitted empirical constant A is approximately 4.31, which is consistent with the theoretical prediction $v(\tau) = Av(0)$.

(3) When the release height is below 30 cm or above 100 cm, the experimental data starts to deviate from the linear relationship, suggesting that the theoretical model may need further refinement under extreme conditions.

The relationship between ejection speed and maximum height, as shown in Figure 4 above:

(1) The experimental data points align well with the theoretical curve derived from equation 5.

(2) The results confirm the validity of the theoretical model that considers air resistance.

(3) In all experiments, when the drop height is 1.2m and the water volume is 340ml, the maximum ejection height observed was 9.625m.

Regarding the fluid dynamic phenomena at the moment of collision, through high-speed photography, the study observed:

(1) At the moment of collision, the water surface exhibited a distinct hemispherical depression.

(2) Subsequently, a high-speed jet was ejected from the center, which corresponds to the phenomenon observed in the "Bokrovski Experiment."

(3) This observation is in high agreement with both the theoretical model and CFD simulation results (Figure 2 above).

The impact of container material, though not the main research focus, was noted:

(1) PVC containers, due to their higher stiffness coefficient, produced more ideal experimental results.

(2) The results suggest that the elastic properties of the container could be an important factor influencing the experimental outcomes, warranting further investigation.

Error analysis:

(1) In most cases, the experimental data aligns well with the theoretical predictions, with relative errors controlled within 5%.

(2) The main sources of error could include: angular deviations of the ping-pong ball in the normal direction (<5°), simplifications in the air resistance model, and unconsidered factors such as surface tension and viscosity.

In summary, the experimental results strongly support the predictions of the theoretical model. The results validate the impact of water volume and release height on ejection height, as well as the relationship between pre- and post-collision speeds. Additionally, the experiments revealed phenomena that merit further study, such as deviations under extreme conditions and the impact of container material. These findings not only confirm the study's understanding of the force transmission mechanisms of water as a medium during collisions but also provide important evidence for further improving the theoretical model.

5. Conclusion

This study, through theoretical analysis, numerical simulations, and systematic experiments, has delved into the force transmission mechanisms caused by the movement of water during collisions when water is used as a medium. A mathematical model incorporating fluid dynamics and air resistance was established, successfully describing the force transmission throughout the entire collision process. CFD simulations and high-speed photography validated the accuracy of the theoretical model, especially the observed high-speed jet phenomenon, which was in strong agreement with the "Bokrovski Experiment" phenomenon.The experimental results show that when the water volume reaches approximately 310 ml, further increases in water volume have minimal effect on the ejection height, validating the reasonableness of the "infinite depth" assumption in the theoretical model. In the release height range of 40-80 cm, the post-collision speed showed a good linear relationship with the initial speed, with the fitted empirical constant being approximately 4.31, consistent with the theoretical prediction. The study also found that the elastic properties of the container material could be an important factor influencing the experimental results, offering new directions for future research.This research not only deepens the understanding of the force transmission mechanisms of water as a medium during collisions but also provides a theoretical foundation for related engineering applications, making a significant contribution to the fields of fluid dynamics and collision mechanics.

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