Construction and Practice of the GGP Blended Teaching Model Based on GeoGebra in Engineering Mathematics

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Abstract: To address the prevalent problems in traditional engineering mathematics teaching — such as high abstraction and a disconnect from practical applications — this paper constructs a GGP blended teaching model that uses the dynamic mathematics software GeoGebra as a unifying thread throughout the entire process. The model systematically divides the teaching process into three progressively advancing stages: G (Guided Pre-class Preparation), G (Driven In-class Exploration), and P (Project-based After-class Application). It aims to reshape the relationship between "teaching" and "learning" by transforming the classroom from a place of knowledge delivery into a training ground for thinking and an incubator for application. This paper elaborates on the theoretical foundations of the model, the specific implementation pathways for each stage, and analyzes its role in fostering students' intuitive perception, scientific inquiry ability, and interdisciplinary innovation and application skills.

***Keywords*:** GeoGebra; GGP Blended Teaching Model; Visualization Teaching; Interdisciplinary Integration

1. Introduction

Engineering mathematics serves as the core foundation of science and engineering disciplines and a key curriculum for cultivating students' logical thinking, data analysis, and complex problem-solving skills. However, traditional teaching models suffer from the drawbacks of "emphasizing theory over application and prioritizing knowledge transmission over ability development," leading to the pedagogical dilemma of "difficult to learn, hard to understand, and challenging to apply."

In response, researchers have explored various directions. In visualization teaching, Chu Pengfei et al. [1] introduced GeoGebra into linear algebra instruction; Qiu Wei [2] explored the visual restructuring of textbooks from the perspective of integrating numbers with shapes. In interdisciplinary teaching, Shi Zhangzhi et al. [3] integrated linear algebra with materials science; Zhang Weifeng et al. [4] proposed a restructured teaching sequence with vector groups and linear spaces as the precursor; Zhang Dong [5] proposed the concept of cross-course integrated teaching; Xie Bing et al. [6] introduced AI applications into engineering mathematics courses. These reforms have significantly enhanced students' learning interest and depth of understanding.

However, existing research largely focuses on reforms within a single course or dimension, with visualization applications remaining at the level of scattered demonstration tools rather than being systematically integrated into the entire teaching process. Based on this, this paper takes "Eigenvalues and Eigenvectors" as an example, integrates constructivist learning theory and visualization cognitive theory, and constructs a GGP blended teaching model spanning pre-class, in-class, and after-class stages.

2. Theoretical Foundations of the GGP Blended Teaching Model

The GGP model is a systematic teaching framework supported by GeoGebra and driven by both problem-based inquiry and project-based application, designed to promote students' cognitive deepening and capability enhancement. Its theoretical foundations include:

Constructivist Learning Theory: This theory holds that knowledge is not acquired through teacher instruction, but is constructed by learners in specific contexts with the help of others and necessary learning materials through the process of meaning-making [7]. The GGP model emphasizes that students actively explore, collaborate, and engage in dialogue within the visual contexts created by GeoGebra, autonomously constructing their understanding of mathematical concepts—a perfect alignment with the core tenets of constructivism.

Visualization Cognitive Theory: Visual thinking plays a crucial role in the cognitive process. Visualization can effectively reduce cognitive load and facilitate the perception and understanding of abstract information [8]. GeoGebra transforms abstract mathematical symbols and operations into dynamic, manipulable graphics and images, providing students with a pathway to "see" and "touch" mathematics, thereby greatly promoting the formation of their intuitive thinking.

Learning by Doing Theory: Educator John Dewey emphasized the importance of learning from practical experience [9]. The project-based application phase (P stage) in the GGP model involves students applying and deepening their knowledge by solving real, complex engineering problems, facilitating their transformation from knowledge consumers to knowledge creators.

3. Practical Pathways of the GGP Blended Teaching Model

3.1 Stage G: Guided Pre-class Preparation

This stage aims to guide students into "thinking ahead" through interactive GeoGebra courseware. The courseware is designed as a "semi-finished product" or "interactive experiment" with guiding questions. Taking "Eigenvalues and Eigenvectors" as an example, the teacher designed a "Deformation Laboratory" courseware: sliders allow real-time adjustment of the transformation matrix elements, displaying a unit circle, grid lines, a draggable vector v and its image Av, along with preset matrices for comparative observation.

3.2 Stage G: Driven In-class Exploration

This stage is the core of the model. The classroom structure is reorganized as follows:

Quick Q&A: The teacher uses GeoGebra to demonstrate and address common issues. For instance, regarding the confusion about "direction invariance," the teacher demonstrates that the standard basis vector $e_1=(1,0)^T$ completely changes direction under a symmetric matrix, clarifying that "direction-invariant vectors" are a special minority.

Core Inquiry: The teacher poses a challenge: For a general n×n square matrix A, which non-zero vectors v satisfy $Av=\lambda v$? Can we find a unified algebraic method? Students work in groups, manipulating different types of transformation matrices, formulating hypotheses, verifying them, and deriving the characteristic equation $\det(A-\lambda I)=0$ from the condition that $(A-\lambda I)v=0$ has non-zero solutions.

Synthesis and Elevation: Each group shares findings, and the teacher creates cognitive conflicts. First, for a 90° rotation matrix, students find no direction-invariant vector in the real domain, leading to complex eigenvalues. Second, for a matrix with a double eigenvalue, students discover they can only find one linearly independent eigenvector, understanding the distinction between algebraic and geometric multiplicity. The class collectively abstracts the strict definitions and geometric meaning of eigenvalues and eigenvectors.

3.3 Stage P: Project-based After-class Application

This stage is the core of the model. Classroom time is primarily devoted to in-depth exploration and thinking training based on pre-class questions. Taking the "Image Scrambling and Period Recovery Prediction Based on the Arnold Cat Map" as an example, the project is carried out in four progressive stages: "Mathematical Modeling—Period Investigation—System Implementation—Extended Analysis."

Mathematical Modeling: An N×N pixel image is discretized into a grid of points, with scrambling abstracted as repeated application of matrix transformation $A=\begin{pmatrix}1 & u\\ v & uv+1\end{pmatrix}$ (mod N). Since $\det A=1$, the transformation is area-preserving. From the characteristic polynomial $\lambda^2-(uv+2)\lambda+1=0$, when $uv>0$, the two eigenvalues are positive real roots that are reciprocals of each other. The image is stretched and mixed along the eigenvector directions, producing a "cat face" scrambling effect.

Period Investigation: Students hypothesize whether the finite grid points can be restored after iterations of A. Using GeoGebra's "Scrambling Laboratory," they observe the "restoration—scrambling—restoration" cycle, predict the recovery period using the relationship between eigenvalues and the Fibonacci sequence (for $u=v=1$, $N=8$, period $T(8)=6$), and verify through computer enumeration.

System Implementation: Students are encouraged to comprehensively use GeoGebra (for parametric demonstration and automatic restoration determination) to create a comparable image scrambling demonstration system.

Extended Analysis: Students are guided to vary the parameters u, v, and the grid size N, compare the scrambling intensity and recovery periods of different matrices, understand the fundamental difference in periodicity between hyperbolic scrambling ($uv+2>2$) and shear-type transformations ($uv+2=2$), and discuss the feasibility and limitations of using this method as an image encryption technique.

The project is carried out in groups over a period of 2–4 weeks, progressing through three stages: "Modeling and Proposal Design," "System Development and Debugging," and "Period Verification and Presentation." The final deliverables may include a project report (with complete mathematical derivation of the scrambling principle and period prediction), a runnable image scrambling program, a GeoGebra demonstration courseware, a demonstration video, or a group defense

presentation. Evaluation employs a four-dimensional standard: "Mathematical Correctness — Depth of Period Prediction — System Completeness — Team Collaboration." The assessment focuses on whether students can clearly explain the mathematical mechanism that "eigenvalues determine the hyperbolic nature of scrambling, and the characteristic polynomial together with the Fibonacci sequence determines the recovery period," and whether they can make reasonable predictions about scrambling intensity and recovery period based on algebraic tools such as eigenvalues and verify them through experiments. This ensures that the project does not remain at the superficial level of "getting the program to run."

4. Discussion and Reflection

The GGP model places higher demands on teachers, who must transform from "lecturers" to "coaches" and "designers." The model also requires institutional support in technology, training, and scheduling. By leveraging GeoGebra as a visualization bridge, the model lowers the cognitive threshold of engineering mathematics and connects mathematics with professional application frontiers, cultivating students' interdisciplinary thinking and innovative abilities.

5. Conclusion

The GGP model systematically constructs a new teaching paradigm centered on students, with visual inquiry as the main thread and project application as the output, providing an effective solution for engineering mathematics reform. Future research may further focus on empirical analysis of the model's impact on students' long-term learning outcomes, critical thinking, and innovative abilities.

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