Habit Formation, Public Capital Congestion Effect and Economic Growth

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Abstract: This paper constructs an endogenous growth model including habit formation and public capital congestion effect, and analyzes the influence of fiscal policy and public capital congestion effect on economic growth under the framework of habit formation. It is found that under decentralized economy, there exists a unique equilibrium growth rate, and that the growth rate increases with the improvement of private production efficiency, public capital production efficiency and the reduction of public capital congestion effect. The influence of habit-forming parameters on the equilibrium growth rate depends on the relative risk aversion coefficient. In the social arrangement economy, habit formation and public capital jointly determine the problem of multiple equilibrium. Similarly, the increase of public capital congestion effect will also reduce the equilibrium economic growth rate.

Keywords: habit formation, public capital, congestion effect, economic growth

1. Introduction
In the traditional Ramsey model, the utility functions of representative individuals are time-separable, which are widely used because of their simplicity. However, subsequent studies have found that the conclusions obtained under the framework of time-divisible utility function are quite different from the real economy. Consumer utility is not only related to the current consumption, and other economic individual consumption affect the economic individual current utility. Therefore, through the development and improvement of Ramsey model, economists use the utility function, introduce the habits inventory, and put forward the theoretical model that can better fit the real economy.

Since the 1980s, economists have gained a deeper understanding of many macroeconomic phenomena by introducing habit formation to explain real economic problems based on the traditional Ramsey model. Lettau and Uhlig [9], Furer [10] and Pagano [11] introduced habit formation to explain the mystery of consumption smoothing; Boldrin et al. [5] introduced habit formation to explain the sensitivity of income growth of consumption; Angelini [3] introduced habit formation to reveal the mystery of excessive growth of consumption; Diaz et al. [8] argued that habit formation could better explain prudent saving. Lettau and Uhlig [9] introduced habit formation into Hansen’s real business cycle economy (RBC) to investigate the impact of exogenous technology shocks on consumption, labor selection, output and investment. They found that habit formation made representative individuals more risk-averse. Christiano et al. [7] introduced habit formation into a new Keynesian framework, and the study found that the dynamic impact of monetary policy shocks on the us economy could be well explained by this model. Carroll et al. [6] examined the economic impact of habit formation under the AK framework. The study found that the conclusions under the habit-forming framework were quite different from those under the time-separable framework. Alonso - Carrera et al. [2] examined the effect of habit formation on the economy in AK economy. The study found that habit formation increases economic growth because it delays consumption and increases savings in the current period.

Extensive literature has proved that the accumulation of public capital by governments can affect private production activities, promote economic growth and increase social welfare. Arrow and Kruz [1] built the exogenous growth models of public capital, and productive public capital was introduced into private production function to promote the economic growth. At the same time, the public capital as a consumer service for utility function affects the private consumption levels. Barro [4], Arrow and Kurz model is a kind of endogenous economic growth model, and presents the optimal public spending, private production, public influence consumer spending and private consumption. These are studied in a time-separable framework, and habit formation, as an important analytical framework of macroeconomics, can explain many economic realities. In this paper, habit formation and public capital stock are introduced into the model to explain the difference of economic growth in reality, and analyze how the crowding effect of public capital, tax rate and the importance parameters of habit formation affect the economic growth rate at equilibrium.
2. Model framework

2.1 The individual

We assume that all individuals in the economy are homogeneous and have a continuous infinite life span. The utility of each individual period is not only determined by current consumption, but also influenced by past consumption. In other words, utility function is introduced in the form of ratio. The form of the utility function is as follows:

$$\max \int_{0}^{\infty} e^{-\theta t} \left(\frac{c}{h^{\gamma}}\right)^{1-\sigma} - \frac{1}{1-\sigma} \, dt$$ (1)

$\theta$ is consumers’ subjective time preference rate, which reflects consumers’ subjective preference for future consumption. The higher of $\theta$, the lower of the discount rate, and the more attention consumers pay to the current consumption. $h$ is the customary stock in the current period; $\gamma (0 \leq \gamma \leq 1)$ reflect the influence intensity of habit stock, when $\gamma = 0$, current utility is only affected by current consumption, when $\gamma = 1$, current utility depends on relative consumption, when $0 < \gamma < 1$, current utility is jointly affected by current consumption and habit stock. $\sigma$ is the relative risk aversion coefficient, and is also the reciprocal of the elasticity of intertemporal substitution.

The expression of habitual stock is:

$$h = \rho \int_{-\infty}^{t} e^{\rho(t-s)} \overline{c}(s) ds$$ (2)

$\rho$ is the adjustment speed of habit formation, which reflects the influence degree of consumption on habit stock in different periods. The larger of $\rho$, the greater the impact of recent consumption is. $\overline{c}(t)$ is the average consumption of the society.

Take the derivative of both sides of equation (2) with respect to time $t$, and then:

$$\dot{h} = \rho(\overline{c}-h)$$ (3)

Budget constraint equation of individuals:

$$\dot{k}_y = (1-\tau)(r k_y + \omega) - c$$ (4)

$r$ is the interest rate, $k_y$ is private capital hold by individuals, $r k_y$ is capital income, $\omega$ is wage income, $\tau$ is income tax rate, and we assume that labor and capital are taxed at the same rate.

The problem of individual optimization is that individuals maximize their lifetime utility function by choosing to consume and save under budgetary constraints, that is, maximize the utility function (1) given equation (4).

We construct Hamilton system to solve the optimization problem:

$$H = \left(\frac{c}{h^\gamma}\right)^{1-\sigma} - \frac{1}{1-\sigma} + \lambda (1-\tau)(r k_y + \omega) - c$$

Where is the co-state variable and represents the shadow price of private capital.

Solve the optimization problem of the above system and obtain the first-order conditions:

$$H_c = \left(\frac{c}{h^\gamma}\right)^{-\sigma} (1/h^\gamma) - \lambda = 0$$ (5)

$$H_{k_y} = \lambda (1-\tau) r = 0 \lambda - \dot{\lambda}$$ (6)
The transverse condition is:
\[
\lim_{t \to \infty} \lambda k_y e^{-\theta t} = 0 \tag{7}
\]

2.2 The manufacturer

Firms produce by employing labor and capital, while productive public expenditure services by governments promote production. The production function of the manufacturer is expressed as (output per capita):
\[
y = Ak^\alpha_y (k^\lambda_g)^{1-\alpha} \tag{8}
\]

Where \( y \) is output per capita; \( A \) denotes production efficiency; \( k_y \) represents the private capital invested in production; \( k^\lambda_g \) represents services provided by productive public capital.

Assuming there are \( n \) individuals in the economy, production-oriented public capital services can be expressed as
\[
k^\lambda_g = k_g (y_g / K)^{1-\mu} \tag{9}
\]

\( k_g \) represents the government’s public capital; \( K \) represents the total private capital; \( \mu \) reflects the congestion of public capital. When \( \mu = 1 \), the public capital services obtained by manufacturers are equal and have nothing to do with the amount of private capital invested, the public services are not exclusive, competitive and crowded. When \( \mu = 0 \), the public capital services provided by the government and the private capital invested by the manufacturers account for the same proportion of the total capital, the public capital services are completely exclusive and competitive, and the public capital services are crowded.

Substituting equation (9) into equation (8), the manufacturer’s production function can be expressed as:
\[
y = Ak^\alpha_y (k^\lambda_g)^{1-\alpha} K^{-(1-\mu)(1-\sigma)} \tag{10}
\]

Firm optimization refers to that, under the conditions of given wages and interest rates, firms maximize profits by selecting labor and private capital, that is, by selecting private capital input to maximize (10) and maximize the results:
\[
r = (1-(1-\alpha)\mu)Ak^\alpha_y k^\lambda_g K^{-(1-\mu)(1-\sigma)} \tag{11}
\]
\[
w = (1-\alpha)\mu Ak^\alpha_y k^\lambda_g K^{-(1-\mu)(1-\sigma)} \tag{12}
\]

It is assumed that there are \( n \) homogeneous manufacturers (individual production) in the economy, and \( n \) remains unchanged. According to the symmetric equilibrium, when \( K = nk_y \), the equilibrium can be obtained. Substitute equations (11) and (12) to obtain:
\[
r = (1-(1-\alpha)\mu)Ak^\alpha_y k^\lambda_g n^{-(1-\mu)(1-\sigma)} \tag{13}
\]
\[
w = (1-\alpha)\mu Ak^\alpha_y k^\lambda_g n^{-(1-\mu)(1-\sigma)} \tag{14}
\]

2.3 The government

The government carries out public capital service expenditure by levying labor income tax and capital income tax. The balanced budget of the government is as follows:
\[
g = \tau (rk_y + \omega) \tag{15}
\]
The right-hand side of the equation is government revenue, and is government productive public capital expenditure. Suppose the accumulation equation of productive public capital of the government is expressed as follows:

\[ \dot{k}_g = Bk_g^\beta g^{1-\beta} \]  

(16)

Here, the government’s public capital stock is introduced into the model. Public capital not only affects the current economic growth, but also affects the future economic growth.

2.4 Competitive equilibrium

The competitive equilibrium of the economy refers to the factor price combination \( \{w, r\} \), policy combination \( \{\tau, g\} \) and other factor combination \( \{c, y, h, k, k_g, k_g^*\} \) existing in the economy that:

1. Family optimization, that is, to meet the budget constraints (4) through the selection of consumption maximization utility (2);
2. Under a given factor price, manufacturers maximize their profits by choosing labor and private capital;
3. Labor and capital market clearing, that is, manufacturers’ demand for labor and capital is equal to individual supply of labor and capital;
4. Balance between family budget constraint and government budget constraint;
5. In equilibrium, the consumption provided by the representative is equal to the average consumption, and the private capital input of the representative manufacturers is equal.

3. Solving for competitive equilibrium

Under the condition that the initial value of the state variable is given, there exists a competitive equilibrium for the variables \( \{c, y, g, h, k, k_g, k_g^*\} \) in the economy: that is, the growth rate of each variable \( \{c, y, g, h, k, k_g, k_g^*\} \) in the economy is the same and equal to the overall growth rate of the economy \( \Omega \).

According to equation (5), it can be obtained that:

\[ \dot{\lambda} / \lambda = -\sigma (\dot{\epsilon} / \epsilon) + \gamma (\sigma - \lambda)(\dot{h} / h) \]  

(17)

Substitute equation (13) into equation (6) to obtain:

\[ \dot{\lambda} / \lambda = 0 - (1-\tau)\theta - (1-\tau)(1-\mu)(1-\alpha)An^{-\theta(1-\lambda)(\alpha)}(k^*_g / k_g)^{\alpha} \]  

(18)

According to equations (17) and (18), it can be obtained that:

\[ ( -\sigma + \gamma (\sigma - \lambda)) \Omega = 0 - (1-\tau)\theta - (1-\tau)(1-\mu)(1-\alpha)An^{-\theta(1-\lambda)(\alpha)}(k^*_g / k_g)^{\alpha} \]  

(19)

Substituting equations (13) and (14) into (15), we can obtain:

\[ g / k_g = \tau An^{-\theta(1-\lambda)(\alpha)}(k^*_g / k_g)^{\alpha} \]  

(20)

According to equation (16):

\[ \dot{k}_g / k_g = B \dot{g} / g^{1-\beta} \]  

(21)

According to equations (20) and (21), it can be obtained that:

\[ \dot{k}_g / k_g = (B/\Omega)^{1/(\alpha(1-\beta))}(\tau A)^{\alpha} n^{-\theta(1-\lambda)(\alpha)/\alpha} \]  

(22)

Substitute equation (22) into equation (19) to obtain:
According to equation (23), the following conclusions can be obtained:

**Proposition 1:** in decentralized economy, we can determine the economic growth rate under the balanced growth path through equation (23).

Equation (23) can determine the economic growth rate at equilibrium, but it cannot obtain the display solution of the growth rate. The left side of the equation increases with the economic growth rate, and the right side decreases with the economic growth rate, so there is a unique intersection to determine the unique equilibrium growth rate in decentralized economy.

**Corollary 1:** when the production efficiency of private sector and government public capital increases, the economic growth rate will increase in equilibrium under decentralized economy.

The curve on the left side of equation (23) is not affected by the production efficiency, while the curve on the right side of the equation moves up with the increase in the production efficiency of the private sector and the government’s public capital. Therefore, it can be intuitively seen that the economic growth rate increases at equilibrium.

**Corollary 2:** when $\tau = 1 - \alpha$, economic growth rate reaches the highest under decentralized economy; When $\tau < 1 - \alpha$, the economic growth rate increases with the increase of tax rate; when $\tau > 1 - \alpha$, the economic growth rate decreases with the increase of tax rate.

The left-hand side of equation (23) is not affected by the tax rate, so let equation (23) differentiate the tax rate left and right, we can get

$$\frac{dRH}{d\tau} = \left[1 - \mu(1 - \alpha)\right]n^{-\left(1 - \mu(1 - \alpha)\right)/\alpha}(B/\Omega)^{1 - \alpha / \alpha}A^{\alpha/(1 - \alpha)}(1 + (1 - \alpha)/(\tau A))$$

When $\tau = 1 - \alpha$, then $\frac{dRH}{d\tau} = 0$; When $\tau < 1 - \alpha$, then $\frac{dRH}{d\tau} = 0$. The curve on the right-hand side of equation (23) is moving, the economic growth rate increases at equilibrium; When $\tau > 1 - \alpha$, then $\frac{dRH}{d\tau} = 0$, the right-hand curve of equation (23) moves down, and the economic growth rate decreases at equilibrium. And income tax rates and economic growth is an inverted U-shaped relationship, income tax rates on the one hand, increase the public accumulation of capital, and capital accumulation has a promoting effect on economic growth, on the other hand the ascension of income tax rate reduces the marginal rate of return of capital, and inhibiting effect to the economic growth and capital accumulation, both role balance, under the optimal rate of economic growth is the largest.

**Corollary 3:** the influence of habit-importance parameter on economic growth rate depends on the relative risk aversion coefficient. When the relative risk aversion coefficient is greater than 1, the economic growth rate increases with the increase of habit-importance degree. Conversely, when the relative risk aversion coefficient is less than 1, the economic growth rate decreases with the increasing importance of habits.

The right side of equation (23) is not affected by habit-importance parameters, and the derivative of the left side of equation (23) with respect to habit-importance parameters is:

$$\frac{dLH}{dy} = \Omega^{-\sigma}, \text{ when } \sigma > 1, \text{ then } \frac{dLH}{dy} < 0, \text{ that is, when the coefficient of relative risk aversion is greater than 1, the importance of habit increases, the left side of equation (23) decreases, and the economic growth rate increases at equilibrium. When } \sigma < 1, \text{ then } \frac{dLH}{dy} > 0, \text{ that is, when the coefficient of relative risk aversion is less than 1, the degree of habitual importance increases, the left side of equation (23) rises, and the economic growth rate declines at equilibrium.}$$

**Corollary 4:** when the number of economic individuals is satisfied $n > e$, the economic growth rate increases as the degree of crowding decreases.

The left side of equation (23) is not affected by the degree of congestion, and the right side takes the derivative of the degree of congestion to obtain:

$$\frac{dRH}{d\mu} = (1 - \tau)(1 - \mu(1 - \alpha))An^{-\left(1 - \mu(1 - \alpha)\right)/\alpha}(B/\Omega)^{1 + \alpha / \alpha}A^{\alpha/(1 - \alpha)}(1 + (1 - \alpha)/(\tau A))$$
When it is satisfied by \( n > e \), that is \( \frac{dRH}{d\mu} > 0 \), when the economic unit is large enough, the economic growth rate increases as the degree of crowding decreases.

**Corollary 5:** when the number of economic individuals increases, the economic growth rate decreases at equilibrium.

The left hand side of equation (23) is unaffected by \( n \) and the right hand side decreases as \( n \) increases, so the economic growth rate declines at equilibrium. This means that, other things being equal, economies with relatively large populations have a relatively low rate of growth in output per head at equilibrium.

### 4. Social planner economy

The social planner economy does not distinguish between families, manufacturers and the government, but maximizes social welfare from the perspective of the whole society.

The optimization problem for social planners is:

\[
\max \int_0^\infty e^{-\sigma t} \left( \frac{(c/h^\gamma)^{1-\sigma}}{1-\sigma} - 1 \right) dt
\]

The constraints for social planners are

\[
\dot{k}_g = \alpha k_g^{1-\alpha} n^{-(1-\mu)(1-\sigma)} - c - g
\]

\[
\dot{h} = \rho(c-h)
\]

\[
\dot{k}_g = Bk_g^{\beta} g^{1-\beta}
\]

We solve the above optimization problem by constructing the following Hamilton function:

\[
H = \frac{(c/h^\gamma)^{1-\sigma}}{1-\sigma} - 1 + \lambda_1 \left( \frac{1}{h^\gamma} \right) - \lambda_2 (1-\beta) B k_g^{\beta} g^{1-\beta}
\]

Where \( \lambda_1, \lambda_2, \lambda_3 \) is the co-state variable, which represents the shadow prices of private capital, public capital and habitual stock respectively.

The first-order conditions for solving the above optimization problem are as follows:

\[
H_c = (c/h^\gamma)^{1-\sigma} (1/h^\gamma) - \lambda_1 + \lambda_3 \rho = 0
\]

\[
H_g = -\lambda_1 + \lambda_2 (1-\beta) B k_g^{\beta} g^{1-\beta} = 0
\]

\[
H_{k_g} = \lambda_1 \alpha A k_g^{\alpha-1} k_g^{1-\alpha} n^{-(1-\mu)(1-\sigma)} = \theta \lambda_1 - \dot{\lambda}_1
\]

\[
H_{h_g} = \lambda_1 (1-\alpha) A k_g^{\alpha-1} k_g^{1-\alpha} n^{-(1-\mu)(1-\sigma)} + \lambda_2 \beta B k_g^{\beta-1} g^{1-\beta} = \theta \lambda_2 - \dot{\lambda}_2
\]

\[
H_{h} = (c/h^\gamma)^{1-\sigma} (1/h^\gamma) - \lambda_3 \rho = \theta \lambda_3 - \dot{\lambda}_3
\]

The transverse condition is:

\[
\lim_{t \to \infty} \lambda_1 k_t e^{-\dot{\theta} t} = \lim_{t \to \infty} \lambda_2 k_t e^{-\dot{\theta} t} = \lim_{t \to \infty} \lambda_3 h e^{-\dot{\theta} t} = 0
\]
According to equation (28):
\[
\lambda_i = \lambda_2 (1 - \beta) B k_g^\beta g^{-\beta} 
\]
\[\text{(33)}\]

\[
\frac{\dot{\lambda}_i}{\dot{\lambda}_1} = 0 - \alpha A (k_y / k_g)^{\alpha - 1} n^{-(1-\mu)(1-\sigma)} 
\]
\[\text{(34)}\]

Substitute equation (28) into equation (30) to obtain
\[
\frac{\dot{\lambda}_2}{\dot{\lambda}_2} = 0 - (1 - \beta)(1 - \alpha) A B (k_y / k_g)^n (k_g / g)^{\beta} n^{-(1-\mu)(1-\sigma)} - \beta B (k_g / g)^{\beta - 1} 
\]
\[\text{(35)}\]

When the economy is in equilibrium, that is, the growth rate of each variable \(\{c, y, g, h, k_y, k_g, k_g^\prime\}\) is the same, and the overall growth rate of the economy is assumed.

According to the above analysis, the equilibrium path is as follows:
\[
\frac{\dot{\lambda}_i}{\dot{\lambda}_1} = \frac{\dot{\lambda}_2}{\dot{\lambda}_2} = \frac{\dot{\lambda}_3}{\dot{\lambda}_3} = \frac{\dot{\lambda}_4}{\dot{\lambda}_4} = \frac{\dot{\sigma}(\gamma - 1) - \gamma}{\Omega} 
\]
\[\text{(36)}\]

According to equations (26), (34), (35) and (36), it can be obtained that:
\[
(\sigma(1-\gamma) + \gamma - \beta) \Omega = (1 - \alpha)(1 - \beta) A B n^{-(1-\mu)(1-\sigma)} (\frac{B}{\Omega})^{\beta - (1-\beta)} \frac{\alpha A}{(\sigma(1-\gamma) + \gamma)} (\frac{\Omega + 0}{\Omega})^{\mu(1-a) - 0} 
\]
\[\text{(37)}\]

The deformation of equation (37) can be obtained as follows:
\[
(\sigma(1-\gamma) + \gamma - \beta) \Omega = (1 - \alpha)(1 - \beta) A B n^{-(1-\mu)(1-\sigma)} (\frac{B}{\Omega})^{\beta - (1-\beta)} \frac{\alpha A}{(\sigma(1-\gamma) + \gamma)} (\frac{\Omega + 0}{\Omega})^{\mu(1-a) - 0} 
\]
\[\text{(38)}\]

**Proposition 2**: the economic growth rate at equilibrium under social arranger economy is determined by equation (38).

**Corollary 7**: when \(\sigma(1-\gamma) + \gamma - \beta > 0\), there is a unique path of balanced growth; When \(\sigma(1-\gamma) + \gamma - \beta < 0\), there may be two paths of balanced growth.

According to equation (38), when \(\sigma(1-\gamma) + \gamma - \beta > 0\), the left side of the equation increases with the economic growth rate, and the right side decreases with the economic growth rate, so there exists the only one economic growth rate under the only social arranger. When \(\sigma(1-\gamma) + \gamma - \beta < 0\), the left side of the equation decreases with the economic growth rate, and the right side also decreases with the economic growth rate, there may be two balanced growth paths in the economy.

**Corollary 8**: if \(\sigma(1-\gamma) + \gamma - \beta > 0\), when the degree of public capital congestion decreases, the economic growth rate increases at equilibrium. When the number of individuals in an economy increases, the rate of economic growth declines at equilibrium. When \(\sigma(1-\gamma) + \gamma - \beta < 0\), the degree of public capital congestion increases, or the number of individuals increases, the difference in economic growth rate between the two possible balanced growth paths will increase.

When \(\sigma(1-\gamma) + \gamma - \beta > 0\), there is a unique equilibrium growth rate in the economy, when the degree of public capital congestion decreases, the right side of equation (37) increases, the left side does not change, and the economic growth rate increases at equilibrium. When the number of individuals in the economy decreases, the right side of equation (37) decreases, while the left side remains unchanged, and the economic growth rate declines at equilibrium. When \(\sigma(1-\gamma) + \gamma - \beta < 0\), the degree of public capital congestion increases or the number of individuals increases, the right side of equation (37) will increase and the left side will remain unchanged, so the difference in economic growth rate between the two possible balanced growth paths will increase. We can also get the above conclusion through simple numerical simulation (as shown in figure 1). In figure 1, increasing n or decreasing will reduce the growth rate on the slower growth path at equilibrium, and the reduction effect is more obvious. It has little effect on the rate of economic growth on the faster path of equilibrium.
Table 1. Parameter selection

<table>
<thead>
<tr>
<th>parameter</th>
<th>A</th>
<th>B</th>
<th>$\sigma$</th>
<th>$\gamma$</th>
<th>$\beta$</th>
<th>$\alpha$</th>
<th>$\theta$</th>
</tr>
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<tbody>
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<td>0.2</td>
<td>0.25</td>
<td>0.8</td>
<td>0.2</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Figure 1. Determination of economic growth rate at equilibrium

5. Conclusion

This paper constructs an economic growth model including habit formation and crowding effect of government public capital expenditure and investigates the path of balanced economic growth under decentralized economy and social planner economy. It is found that there is a unique balanced growth path in decentralized economy. Moreover, the economic growth rate increases with the improvement of private production efficiency and government public capital production efficiency. The effect of habit-forming importance parameter on economic growth depends on the relative risk aversion coefficient. The optimal tax rate depends on the output elasticity of private capital; less crowded public capital will increase economic growth; an increase in the number of individuals in an economy lowers the rate of economic growth.

Arranged in society economy, the habit formation and whether the government public capital productivity determines the equilibrium has uniqueness. When the economy exists in the two equilibrium paths, public capital congestion increases or the number of individuals in the economy increase will reduce the slower equilibrium path of economic growth and have less effect on the rapid equilibrium path of growth.

In the further expansion of this paper, we introduce the elasticity of substitution of public capital and private capital to see what impact it has on the structure, or introduce habit formation in the quantitative analysis of the impact of tax policies on economic growth.
Reference


