# Can Volatility Risk Premia Improve Fama-French Three-Factor Model in the Explanation of Stock Index Returns? — Evidence from US Industrial Stock Indices

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**Abstract:** In this dissertation, the author studies the volatility risk premia (i.e., the difference between implied volatility and realized volatility), which is documented in recent literature as a risk factor to explain equity returns. The author empirically tests the Fama-French three-factor model and the multifactor asset pricing model involving volatility risk premia in three industrial market index returns in the US market – motorcar, IT and banking industries. It is found that adding volatility risk premia to Fama-French model can improve the explanation of cross-sectional stock index returns. This finding is consistent with Bollerslev et al.[1] in which volatility risk premia consistently explains stock returns.
***Keywords*:** volatility risk premia, fama-french three-factor model, stock index returns

## 1. Introduction

Is the return on the stock market predictable? This question has been debated in theoretical and empirical studies for a long time with various findings. For instance, The Capital Asset Pricing Model (CAPM hereafter) use market portfolio return to explain stock return. However, the validity of CAPM has been challenged since it was introduced as empirical evidence shows that CAPM fails to capture some other sources of systematic risk. Fama and French[2] introduce size and book-to-market ratio risk factor into CAPM. The Fama-French three-factor model (FF, hereafter) is an empirical model based on the Arbitrage Pricing Theory (APT). According to APT, there are various sources of systematic risk should be considered in asset pricing, where market portfolio return (the one used in CAPM) is not the only one factor capturing the whole systematic risk. However, APT does not identify the exact sources of systematic risk, giving rooms to improve the multifactor asset pricing model, such as FF.

The aim of this dissertation is to compare the FF model with the other multifactor model involving volatility risk premia ($IV_t - RV_t$). The author wants to investigate if adding ($IV_t - RV_t$) can improve FF model in the explanation of stock index return in industrial level. The risk factors taken into account include: (1) the excess market index return, $Rm - Rf$. (2) Small stock return minus big stock return, $SMB_t$. (3) the high book-to-market ratio stock returns minus the low book-to-market ratio stock returns, $HML_t$. (4) realized volatility, $RV_t$, (5) implied volatility, $IV_t$, and (6) the volatility risk premia, ($IV_t - RV_t$).

The author investigates whether the volatility risk premia can explain industrial index returns with comparison to FF model in three US industrial index returns – motorcar, IT and banking. It's found that adding volatility risk premia to FF model can improve the explanation of stock index returns in terms of adjusted $R^2$ s in three industries. Besides, including realized volatility to FF model increases adjusted $R^2$ s as well. The findings confirm that there are other risk factors can explain stock returns apart from FF three factors, in particular, volatility risk premia has consistent performance to explain stock returns.

## 2. Literature review

### 2.1 The CAPM model

Jack Treynor[3], John Lintner[4] and Jan Mossin[5] advanced the CAPM model. It works as follows:

$$E(r_i) = r_f + B_{im}\left(E(r_m) - r_f\right) \tag{i}$$

Where $E(r_i)$ is the expected return of the capital asset. $r_f$ is the risk-free rate of interest. $\beta_{im}$ is the systematic market risk. $E(r_m)$ is the expected return of the market. $(E(r_m) - r_f)$ is the expected market rate of return minus the risk-free rate of return (market premium).

## 2.2 The Fama French three-factor model

Since more and more scholars criticized the SLB CAPM model, such as Roll and Ross[6], who found that beta is not the sole determinant of risk. Lakonishok and Shapiro[7] pointed out that the return rate will be influenced by the unsystematic risks and Fama French[8] proved that there isn't significant relationship between beta and the cross-section of average returns. Therefore, for the past few years, Fama French three-factor model has been widely used since it has increased the explanatory power of the returns. The FF model is based on the Aribitrage Pricing Theory (APT) which is propounded by Ross[9].

The FF model works as follows:

$$r_t = \alpha + \beta\left[\left(r_m - r_f\right)_t, SMB_t, HML_t\right] + \varepsilon_t \tag{ii}$$

As we can see, FF shows that the risk premium is related to three factors which are (1) the expected market rate of return minus the risk-free rate of return (market premium), (2) small market capitalization minus big market capitalization ($SMB_t$) and (3) the high book-to-market ratio minus the low book-to-market ratio ($HML_t$).

# 3. Model

To explore the improvement of volatility risk premia in the Fama French three-factor model for the explanation of cross-sectional industrial stock returns in the US market, four models will be implemented in this section. Firstly, the standard three-factor model is used as the baseline to be compared. Then, we import IVt and RVt in the model separately to investigate whether they have statistical significance on the industrial stock returns. Lastly, the variable (IVt - RVt) is introduced in the three-factor model under the prerequisite that both of the variables have effects on the dependent variable. The software utilized to this problem is Stata which widely used for statistical data analysis in the various field.

The four regression models are:

$$R_t = \alpha + \beta[Rm - Rf)_t, SMB_t, HML_t] + \varepsilon_t \tag{1}$$

$$R_t = \lambda + \gamma'\left[\left(Rm - Rf\right)_t, SMB_t, HML_t, \left(IV_t - RV_t\right)\right] + \mu_t \tag{2}$$

$$R_t = \lambda + \gamma'\left[\left(Rm - Rf\right)_t, SMB_t, HML_t, RV_t\right] + \mu_t \tag{3}$$

$$R_t = \lambda + \gamma'\left[\left(Rm - Rf\right)_t, SMB_t, HML_t, IV_t\right] + \mu_t \tag{4}$$

where

$R_t$ : the industrial index excess return, i.e., the industrial index return minus the US 3-month Tbill rate.

$Rm - Rf$ : the S&P500 index excess return, i.e., the S&P 500 index return minus the US 3-month Tbill rate.

$SMB_t$ : the Fama-French size factor.

$HML_t$ : the Fama-French book-to-market factor.

($IV_t - RV_t$): the volatility risk premia.

$IV_t$: the VIX index, a proxy of implied volatility of S&P500.

$RV_t$: the realized volatility of S&P500. It is the sum of the squared daily returns over one period t: $RV = \frac{1}{n_t}\sum_{l=1}^{n_t} r_l^2 \times 252$.

Where $r_l = \log\left(S_l / S_{l-1}\right)$ is the closing price of market index on ℓth trading day of month t, and $n_t$ is the number of trading days in month t. The RV is annualized by 252 trading days.

All the variables in the four models are annualized.

As we can see, the model (1) is the original Fama French three-factor model. The three others are the models in which we separately put the $IV_t$, $RV_t$ and $IV_t - RV_t$ into the original model to form a new model. We expect that the prediction of volatility risk premia in the FF model for the explanation of cross-sectional industrial stock returns can be improved.

We assume the three new models can improve the traditional Fama French model at some extent. And then, the value of Adjusted R-square in each model is used as the index which gives us the information of improvement. Finally, the model which has the highest Adjusted R-square will be treated as the best model.

## 4. Data

According to the description of the model in the last section, we need to obtain $R_t$, $Rm - Rf$, $SMB_t$, $HML_t$, $IV_t$, $RV_t$ for the sake of analyzing. The data used for empirical analysis are the time series of 215 months, ranging from January 2000 to December 2012 and involving S&P500 automobiles, S&P500 banks and S&P500 IT industries.

More specifically, the monthly data of total return index $R_t$ is calculated by the formula:

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right) * 100 - R_{ft} \tag{iii}$$

Where the proxy for risk free return $R_{ft}$ is the one-month Treasury bill rate from Fama-French data library, $P_t$ is the price index from the Datastream of University of Essex. It should be noted that the number of the sample turns to 214 since $R_t$ should be calculated by the lag of the price index.

The total return index of automobiles, IT industries and banks (car_rf, bank_rf and it_rf) are the dependent variables of the model, which have the characteristics as shown in the table below.

**Table 1: The summary of total return index in three industries**

| Variable | Obs | Mean | Std. Dev. | Min | Max |
|---|---|---|---|---|---|
| car_rf | 214 | -.2788059 | 10.81975 | -59.46552 | 59.52686 |
| bank_rf | 214 | -.0733571 | 8.306055 | -45.50499 | 21.616 |
| it_rf | 214 | -.0885885 | 10.76104 | -55.07973 | 20.45957 |

The three factors of Fama-French model including $Rm - Rf$, $SMB_t$ and $HML_t$ are downloaded from the data library of Kenneth R. French, which is a website containing the Benchmark Returns for month quarter and year, with historical returns back to 1926.

The Implied Volatility of the market portfolio at month t ($IV_t$) is from the CBOE website, which is a measure of the expected volatility calculated as 100 times the square root of the expected 30-day variance (var) of the S&P 500 rate of return as said on its website. It can be written as follows:

$$Volatility\ Index = 100\sqrt{\frac{365}{30} \times \sum_{i=1}^{30} \sigma_i^2}$$ , where $\sigma_i^2$ the S&P 500 rate of return in time i.

In terms of $RV_t$ (Realized Volatility), the data available is given on daily and weekly format. But what we need to analyze in this thesis should be by monthly, hence, it is necessary to do some calculations on the original data. Carr and Wu[10] use the sum of the squared daily returns during one period to calculate the monthly Realized Volatility. The equation is:

$$RV_t = \frac{1}{n_t} \sum_{l=1}^{n_t} r_l^2 \times 252$$ ,Where $RV_t$ is the Realized Volatility on month t, $n_t$ is the total days of trading in month t. They also gave the conclusion that there is no difference from how to calculate the total days of a year. Both the actual days 365 and the business days 252 can be used. In this article, we use the actual days 365 instead. The summary of this variables are list in Table 2.

**Table 2: The summary of realized volatility in the three industries**

| Variable | Obs | Mean | Std. Dev. | Min | Max |
|---|---|---|---|---|---|
| car_rvol | 214 | 36.27415 | 21.19377 | 12.01027 | 159.8856 |
| bank_rvol | 214 | 31.65821 | 26.95641 | 7.984671 | 169.8559 |
| it_rvol | 214 | 39.79348 | 22.26967 | 8.900942 | 197.181 |

## 5. Empirical Analysis

The four models mentioned before will be utilized in this chapter to do the empirical analysis. Three subsections are included to correspond to the three industries analyzed. In each part, the four models will be written based on the data described on the last chapter. Then, the F-values, R-squares and Adjusted R-squares are listed in the table to make a comparison of the models. As a result, we will find out which model improves the FF model best in each case.

## 5.1 The result of car industry

For the classic three-factor model:

$$R_t = -0.8038 + 0.3603(Rm - Rf)_t + 0.3648SMB_t + 0.9518HML_t + \varepsilon_t$$
$$(0.7214) \quad (0.1598) \quad (0.2156) \quad (0.2267)$$

$$\text{Adj } R^2 = 0.07$$

**Table 3: The result of regression for Fama French model in automobiles**

| car_rf | Coef. | Std. Err. | t | P>\|t\| | [95% Conf. Interval] | |
|---|---|---|---|---|---|---|
| mkt_rf | .3603418 | .1598048 | 2.25 | 0.025 | .0453147 | .6753689 |
| smb | .3647921 | .2156082 | 1.69 | 0.092 | -.0602417 | .7898259 |
| hml | .9518013 | .2267186 | 4.20 | 0.000 | .5048653 | 1.398737 |
| _cons | -.8038264 | .7214129 | -1.11 | 0.266 | -2.225965 | .6183126 |

For the new model with variable $(IV_t - RV_t)$

We can obtain the model below from Stata, which shows a significant positive estimate for each coefficient from January 2000 to December 2012 in the field of automobiles.

$$R_t = 1.4020 + 0.2894(Rm - Rf)_t + 0.3293SMB_t + 0.8729HML_t + 0.1461(IV - RV)_t + \varepsilon_t$$
$$(0.9679) \quad (0.1576) \quad (0.2109) \quad (0.2227) \quad (0.0440)$$

$$\text{Adj } R^2 = 0.12$$

**Table 4: The result of regression for model with variable ($IV_t - RV_t$) in automobiles**

| car_rf | Coef. | Std. Err. | T | P>\|t\| | [95% Conf. Interval] | |
|---|---|---|---|---|---|---|
| mkt_rf | 0.289438 | 0.157562 | 1.84 | 0.0680 | -.0211771 | .6000528 |
| smb | 0.329258 | 0.210897 | 1.56 | 0.1200 | -.0864998 | .7450166 |
| hml | 0.872898 | 0.222748 | 3.92 | 0.0000 | .4337776 | 1.312018 |
| car_vrp | 0.146127 | 0.043955 | 3.32 | 0.0010 | .0594756 | .2327787 |
| _cons | 1.401974 | 0.967934 | 1.45 | 0.1490 | -.5061909 | 3.310138 |

For the new model with variable $RV_t$

$$R_t = 3.5328 + 0.2173(Rm - Rf)_t + 0.3186SMB_t + 0.8238HML_t - 0.1162RV_t + \varepsilon_t$$
$$(1.4617) \quad (0.1616) \quad (0.2109) \quad (0.2245) \quad (0.0343)$$

$$\text{Adj } R^2 = 0.12$$

**Table 5: The result of regression for model with variable RVt in automobiles**

| car_rf | Coef. | Std. Err. | t | P>\|t\| | [95% Conf. Interval] | |
|---|---|---|---|---|---|---|
| mkt_rf | 0.217349 | 0.161586 | 1.35 | 0.1800 | -.1011976 | .5358953 |
| smb | 0.318637 | 0.210872 | 1.51 | 0.1320 | -.0970722 | .7343452 |
| hml | 0.823801 | 0.224482 | 3.67 | 0.0000 | -0.1838854 | -0.0485395 |
| car_rvol | -0.11621 | 0.034328 | -3.39 | 0.0010 | -0.1838854 | -0.2324249 |
| _cons | 3.532813 | 1.461738 | 2.42 | 0.0170 | .6511731 | 6.414453 |

For the new model with variable $IV_t$

$$R_t = 3.9209 + 0.2019(Rm - Rf)_t + 0.3320SMB_t + 0.8324HML_t - 0.2126IV_t + \varepsilon_t$$
$$(2.2918) \quad (0.1744) \quad (0.2143) \quad (0.2314) \quad (0.0980)$$

Adj $R^2 = 0.09$

**Table 6: The result of regression for model with variable IVt in automobiles**

| car_rf | Coef. | Std. Err. | t | P>\|t\| | [95% Conf. Interval] |
|---|---|---|---|---|---|
| mkt_rf | 0.201892 | 0.174432 | 1.16 | 0.2480 | -.1419796 .5457639 |
| smb | 0.33205 | 0.214261 | 1.55 | 0.1230 | -.0903409 .7544398 |
| hml | 0.832422 | 0.231379 | 3.6 | 0.0000 | .3762867 1.288557 |
| iv | -0.21262 | 0.097986 | -2.17 | 0.0310 | -0.4057877 -0.0194528 |
| _cons | 3.920897 | 2.291815 | 1.71 | 0.0890 | -.5971399 8.438934 |

Comparison of the models

**Table 7: The comparison of the model in automobiles**

| Method | Model 1 | Model 2 | Model 3 | Model 4 |
|---|---|---|---|---|
| F value | 6.90 | 8.19 | 8.30 | 6.45 |
| Prob>F | 0.0002 | 0.0000 | 0.0000 | 0.0001 |
| Root MSE | 10.396 | 10.156 | 10.146 | 10.306 |
| R-square | 0.0898 | 0.1355 | 0.1371 | 0.1098 |
| Adjusted R-square | 0.0768 | 0.1190 | 0.1206 | 0.0928 |

## 5.2 The result of the bank industry

For the classic three factor model:

$$R_t = -0.3301 + 0.1447(Rm - Rf)_t + 0.3485SMB_t + 0.3792HML_t + \varepsilon_t$$
$$(0.5697) \quad (0.1262) \quad (0.1703) \quad (0.1790)$$

Adj $R^2$= 0.02

**Table 8: The result of regression for Fama French model in bank industry**

| bank_rf | Coef. | Std. Err. | t | P>\|t\| | [95% Conf. Interval] |
|---|---|---|---|---|---|
| mkt_rf | 0.144695 | 0.126194 | 1.15 | 0.0530 | -.104075 .3934652 |
| smb | 0.348453 | 0.170261 | 2.05 | 0.0420 | .012813 .6840931 |
| hml | 0.379184 | 0.179035 | 2.12 | 0.0350 | .0262487 .7321201 |
| _cons | -0.33013 | 0.569684 | -0.58 | 0.5630 | -1.453162 .7929031 |

For the new model with variable (IVt - RVt)

$$R_t = 0.7245 + 0.0939(Rm - Rf)_t + 0.3239SMB_t + 0.2905HML_t + 0.0997(IV_t - RV_t) + \varepsilon_t$$
$$(0.6130) \quad (0.1228) \quad (0.1648) \quad (0.1747) \quad (0.0254)$$

Adj $R^2$= 0.09

**Table 9: The result of regression for model with variable (IVt - RVt) in bank industry**

| bank_rf | Coef. | Std. Err. | t | P>\|t\| | [95% Conf. Interval] |
|---|---|---|---|---|---|
| mkt_rf | 0.093913 | 0.122755 | 0.77 | 0.4450 | -.148084 .3359104 |
| smb | 0.323888 | 0.164818 | 1.97 | 0.0510 | -.0010319 .6488072 |
| hml | 0.290526 | 0.174652 | 1.66 | 0.0980 | -.0537795 .6348312 |
| bank_vrp | 0.099654 | 0.025376 | 3.93 | 0.0000 | .0496279 .14968 |
| _cons | 0.724537 | 0.613034 | 1.18 | 0.2390 | -.4839852 1.933059 |

For the new model with variable RVt

$$R_t = 2.5579 + 0.0342(Rm - Rf)_t + 0.3132SMB_t + 0.2514HML_t - 0.0880IV_t + \varepsilon_t$$
$$(0.8735) \quad (0.1241) \qquad (0.1640) \quad (0.1748) \quad (0.0207)$$

Adj $R^2$= 0.10

**Table 10: The result of regression for model with variable $RV_t$ in bank industry**

| bank_rf | Coef. | Std. Err. | t | P>\|t\| | [95% Conf. Interval] |
|---|---|---|---|---|---|
| mkt_rf | 0.034227 | 0.124129 | 2.28 | 0.0830 | -.2104783 .2789319 |
| smb | 0.313194 | 0.163964 | 1.91 | 0.0570 | -.0100405 .6364292 |
| hml | 0.251432 | 0.174801 | 1.44 | 0.0520 | -.0931672 .596032 |
| bank_rvol | -0.08804 | 0.020737 | -4.25 | 0.0000 | -0.1289166 -0.0471558 |
| _cons | 2.557871 | 0.873482 | 2.93 | 0.0040 | .8359055 4.279836 |

For the new model with variable $IV_t$

$$R_t = 5.7037 - 0.0577(Rm - Rf)_t + 0.3066SMB_t + 0.2267HML_t - 0.2715IV_t + \varepsilon_t$$
$$(1.7766) \quad (0.1352) \qquad (0.1661) \quad (0.1793) \quad (0.0760)$$

Adj $R^2$= 0.08

**Table 11: The result of regression for model with variable $IV_t$ in bank industry**

| bank_rf | Coef. | Std. Err. | t | P>\|t\| | [95% Conf. Interval] |
|---|---|---|---|---|---|
| mkt_rf | -0.05766 | 0.135216 | -0.43 | 0.6700 | -.3242176 .208905 |
| smb | 0.306638 | 0.16609 | 1.85 | 0.0660 | -.0207886 .6340654 |
| hml | 0.226729 | 0.179359 | 1.26 | 0.2080 | -.1268567 .5803139 |
| iv | -0.27153 | 0.075956 | -3.57 | 0.0000 | -0.4212699 -0.1217922 |
| _cons | 5.703675 | 1.776561 | 3.21 | 0.0020 | 2.2014 9.205951 |

Comparison of the models

**Table 12: The comparison of the model in bank industry**

| Method | Model 1 | Model 2 | Model 3 | Model 4 |
|---|---|---|---|---|
| F value | 2.68 | 6.00 | 6.68 | 5.32 |
| Prob>F | 0.0480 | 0.0001 | 0.0000 | 0.0004 |
| Root MSE | 8.2096 | 7.9414 | 7.8958 | 7.9886 |
| R-square | 0.0369 | 0.1030 | 0.1133 | 0.0924 |
| Adjusted R-square | 0.0231 | 0.0859 | 0.0963 | 0.0750 |

According to the R-squares of each model, the fitness of model 3 is the best, since it can explain 11.33% variation for the bank industry returns. Correspondingly, the adjusted R-square is the largest one of all as well. The conclusion that model

3 is the best one can also be made from the Root MSE, which represents the root of mean standard error of the model, being the smallest one (with a value of 7.8958).

According to the adjusted R-squares, the model with ($IV_t$ – $RV_t$) and the model with IVt are the two best models among the four. They improve the adjusted R-square of FF model from 0.2 to 0.859 and 0.2 to 0.0963 respectively. That means the model with ($IV_t$ – $RV_t$) improve the explanation of the multifactor model to stock index return.

## 5.3 The result of the IT industry

For the classic three-factor model:

$$R_t = -0.4822 + 0.1059(Rm - Rf)_t + 0.6947SMB_t + 0.6807HML_t + \varepsilon_t$$
$$(0.7263) \quad (0.1609) \quad (0.2171) \quad (0.2283)$$

Adj $R^2$= 0.05

**Table 13: The result of regression for Fama-French model in IT industry**

| it_rf | Coef. | Std. Err. | t | P>\|t\| | [95% Conf. Interval] |
|---|---|---|---|---|---|
| mkt_rf | 0.105942 | 0.160889 | 0.66 | 0.0110 | -.2112226 .4231061 |
| smb | 0.694653 | 0.217071 | 3.2 | 0.0020 | .2667356 1.12257 |
| hml | 0.680669 | 0.228257 | 2.98 | 0.0030 | .2307003 1.130637 |
| _cons | -0.48222 | 0.726307 | -0.66 | 0.5070 | -1.914007 .9495679 |

For the new model with variable ($IV_t$ – $RV_t$)

$$R_t = 1.0195 + 0.0273(Rm - Rf)_t + 0.6869SMB_t + 0.6542HML_t + 0.0799(IV_t - RV_t) + \varepsilon_t$$
$$(0.9955) \quad (0.1635) \quad (0.2152) \quad (0.2266) \quad (0.0366)$$

Adj $R^2$= 0.07

**Table 14: The result of regression for model with variable (IVt - RVt) in IT industry**

| it_rf | Coef. | Std. Err. | t | P>\|t\| | [95% Conf. Interval] |
|---|---|---|---|---|---|
| mkt_rf | 0.02728 | 0.163481 | 0.17 | 0.0680 | -.2950024 .349562 |
| smb | 0.686929 | 0.215177 | 3.19 | 0.0020 | .2627332 1.111126 |
| hml | 0.654156 | 0.22656 | 2.89 | 0.0040 | .207519 1.100792 |
| it_vrp | 0.07989 | 0.036579 | 2.18 | 0.0300 | .0077801 .1520005 |
| _cons | 1.019504 | 0.995481 | 1.02 | 0.3070 | -.9429669 2.981975 |

For the new model with variable $RV_t$

$$R_t = 2.8987 - 0.0366(Rm - Rf)_t + 0.6740SMB_t + 0.6070HML_t - 0.0824RV_t + \varepsilon_t$$
$$(1.5747) \quad (0.1697) \quad (0.2148) \quad (0.2277) \quad (0.0342)$$

Adj $R^2$= 0.08

**Table 15: The result of regression for model with variable RVt in IT industry**

| it_rf | Coef. | Std. Err. | t | P>\|t\| | [95% Conf. Interval] |
|---|---|---|---|---|---|
| mkt_rf | -0.03664 | 0.169697 | -0.22 | 0.8290 | -.3711753 .2978993 |
| smb | 0.673992 | 0.214793 | 3.14 | 0.0020 | .2505541 1.097429 |
| hml | 0.607037 | 0.227736 | 2.67 | 0.0080 | .1580837 1.05599 |
| it_rvol | -0.08242 | 0.034164 | -2.41 | 0.0170 | -0.149774 -0.0150726 |
| _cons | 2.89868 | 1.574654 | 1.84 | 0.0670 | -.2055608 6.00292 |

For the new model with variable $IV_t$

$$R_t = 1.8012 + 0.0294(Rm - Rf)_t + 0.6788SMB_t + 0.6230HML_t - 0.1028IV_t + \varepsilon_t$$

(2.3272) (0.1771) (0.2176) (0.2350) (0.0995)

Adj $R^2$= 0.05

**Table 16: The result of regression for model with variable $IV_t$ in IT industry**

| it_rf | Coef. | Std. Err. | t | P>\|t\| | [95% Conf. Interval] |
|---|---|---|---|---|---|
| mkt_rf | 0.029363 | 0.177131 | 0.17 | 0.0680 | -.31983 .3785558 |
| Smb | 0.678828 | 0.217577 | 3.12 | 0.0020 | .2499019 1.107755 |
| hml | 0.622972 | 0.234959 | 2.65 | 0.0090 | .1597788 1.086166 |
| iv | -0.10276 | 0.099502 | -1.03 | 0.3030 | -.2989162 .093397 |
| _cons | 1.801245 | 2.327279 | 0.77 | 0.4400 | -2.786706 6.389195 |

Comparison of the models

**Table 17: The comparison of the model in IT industry**

| Method | Model 1 | Model 2 | Model 3 | Model 4 |
|---|---|---|---|---|
| F value | 5.05 | 5.05 | 5.33 | 4.06 |
| Prob>F | 0.0021 | 0.0007 | 0.0004 | 0.0035 |
| Root MSE | 10.467 | 10.374 | 10.349 | 10.465 |
| R-square | 0.0673 | 0.0881 | 0.0926 | 0.0720 |
| Adjusted R-square | 0.0540 | 0.0707 | 0.0752 | 0.0543 |

Regression on the data of the IT industry gives the conclusion that model 3 is the best one of the four, since its adjusted R-square is 0.0752 which fits the data of $R_t$ better than the others. From the row of the F-test p-value, model 3 has the smallest one which is 0.0004. That means the joint significance of model 3 is the best compared with the other models, therefore we have evidence to reject the null hypothesis that there is no relationship between the independent variables and the dependent variable. As a consequence, the Volatility Risk Premia improve Fama-French model in the Explanation of Stock Index Returns in the IT industry.

## 6. Conclusion

In principle, the empirical evidence shows that no matter which new factor we put into the Fama French three-factor model, each of them can improve the prediction of expected returns. But generally speaking, in three industries, the model which is added $RV_t$ will be better to explain the expected stock return.

All in all, after having tested the data wihin this 4 models, this thesis provided evidence to show that, $IV_t$, $RV_t$ and ($IV_t$ – $RV_t$) can improve the prediction of Fama French three-factor model. However, there is still more work to be done in the future. Because the data we test is only in US stock market, and we only test in three industries.

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