# Research on the Hedge Ratio of China's Crude Oil Futures — Based on DCC-GARCH Model

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**Abstract:** Crude oil plays an important role in economic development. This paper chooses China's crude oil futures and crude oil actuals as the research objects, and builds the DCC-GARCH model to study the hedge ratio under the risk minimization standard. The hedge ratios obtained from the DCC-GARCH model will be compared with those obtained from OLS, B-VAR and VECM models. The empirical results prove that: China's crude oil futures and actuals have a significant reverse "leverage effect"; China's crude oil futures have a variance reduction of more than 70% under all models; the DCC-GARCH model achieves the best hedging performance in the four models.
***Keywords*:** crude oil futures, DCC-GARCH, hedge ratio

## 1. Introduction

Petroleum is the foundation that supports the development of the entire industrial system. It can be used as a fuel as well as a raw material for the chemical industry. Petroleum is obtained by refining crude oil. Therefore, crude oil has a key impact on the development of China's industrial economy. Today, the annual trading volume of West Texas Light Crude (WTI) and North Sea Brent (Brent) crude oil in the United States is huge, and they are the mainstream of many international crude oil varieties. When pricing other crude oils, the prices of these two crude oils must be referenced. Today, China's economy has changed from rapid development to high-quality development, but the demand for fossil fuels such as petroleum is still huge. Before 2018, China did not have its own crude oil futures, and its power in the international crude oil market was relatively weak. Domestic crude oil production and processing enterprises can only face the huge risks brought by oil price fluctuations, or use foreign crude oil futures for hedging while assuming certain costs and risks. On March 26, 2018, China had its first crude oil futures contract, the INE crude oil futures contract. Since then, companies in the crude oil production and processing industry chain in China have been able to hedge their risks through domestic crude oil futures at a lower cost.

However, compared with other countries, the development time of China's crude oil futures is only 3 years, which is very short, and there are few domestic studies related to it. Therefore, this paper uses the existing INE crude oil futures data and Daqing crude oil actuals data to construct a dynamic conditional correlation coefficient (DCC) GARCH model, and then uses this model to estimate the optimal hedge ratio and its performance. This can improve the theoretical system of domestic hedge ratio research, and at the same time provide suggestions for investors participating in the futures market to better hedge risks.

## 2. Literature review

Ederington (1979) chose the least square method to replace the traditional hedging strategy with a 1:1 ratio of futures to actuals positions, and used the variance reduction ratio to evaluate hedging performance. Since then, the study found that the lag term in the variable will have an impact on the current variable, that is, autocorrelation. In order to eliminate this effect, the autoregressive (AR) term is added to the regression equation. Myers (1989) proved through empirical research that the bivariate vector autoregressive model (Bivariate-VAR) is more effective than the OLS model. On the basis of autocorrelation, Ghosh (1993) and Lien (1996) considered the concept of cointegration in the VAR model and used the Vector Error Correction Model (VECM) to calculate the hedge ratio. However, with the development of measurement theory, some studies have found that the fluctuations of asset sequence prices or yields are clustered, that is, violent fluctuations and weak fluctuations tend to occur together in the same time period. In order to better characterize this phenomenon, Engle (1982) pioneered the Autoregressive Conditional Heteroscedasticity (ARCH) model. After that, Bollerslev (1986) extended the ARCH model and proposed a generalized autoregressive conditional heteroscedasticity (GARCH) model. The GARCH model is very accurate in characterizing variables in the aspect of "volatility agglomeration", which has led many scholars to introduce it into the study of hedge ratios. In the study of Tae et al. (1995), the ECM model was used to capture the cointegration relationship between futures and actuals returns, and the GARCH model was used to fit the characteristics of the series of actuals returns. Yang (2005) used binary GARCH models such as BEKK-GARCH and DCC-GARCH to improve the original hedging model, and found that this improvement has a good effect.

Although domestic research on hedging strategies has fallen slightly abroad, it has continued for decades. In the early stage of the research, Hua Junzhou et al. (2003) used the minimum variance hedging method to study the hedging function of China's copper futures. Yuan Xiang and Cao Fanyu (2003) used the error correction model to describe the co-integration relationship, and used the GARCH model to fit the fluctuations of stock index futures and actuals yields, and achieved good results. Yuan Chen and Fu Qiang (2017) used domestic stock index futures data to construct binary GARCH family models such as CCC-GARCH and DCC-GARCH. Yang Jie and Guo Junfeng (2017) compared the DCC-GARCH model with the VECM model and found that the effects of the dynamic model and the static model are almost the same. Zhao Shuran et al. (2016) combined the ECM model and the DCC model and took CVaR as the optimization goal to hedge. The empirical results show that the ECM-DCC model is better than the ECM-CCC model. From the perspective of model reset, Fu Jianru et al. (2019) selected binary GARCH family models such as DCC-GARCH to study the hedging of China's stock index futures, and their results showed that the hedging efficiency after the model reset was higher. Song Bo and Xing Tiancai (2020) compared the DCC-GARCH model with the state-space model and once again proved that DCC-GARCH has a wide range of applicability in the field of hedging.

Throughout the past and present, most domestic researches on the hedging function have selected stock index futures. This is because China only launched crude oil futures in 2018, so there are few domestic studies on its hedging function. At the same time, DCC-GARCH has been widely used in hedging research due to its advantages in accurately portraying the "volatility aggregation" characteristics of asset prices or return rate sequences. Therefore, this paper chooses the DCC-GARCH model to calculate the optimal hedge ratio of China’s crude oil futures to spot, and compares it with the OLS, B-VAR and ECM models.

# 3. Model building

## 3.1 DCC-GARCH model

Engle (2002) proposed a Dynamic Conditional Correlation (DCC) model based on the CCC-GARCH model. The model equation is as follows:

$$\left(r_t \mid \Omega_{t-1}\right) \sim N\left(0, D_t R_t D_t\right) \tag{1}$$

$$D_t^2 = diag\left\{\omega_i\right\} + diag\left\{k_i\right\} \circ r_{t-1} r_{t-1}' + diag\left\{\lambda_i\right\} \circ D_{t-1}^2 \tag{2}$$

$$\varepsilon_t = D_t^{-1} r_t \tag{3}$$

$$Q_t = \overline{Q}(1-\alpha-\beta) + \alpha(\varepsilon_{t-1}\ \varepsilon_{t-1}') + \beta Q_{t-1} \tag{4}$$

$$R_t = diag\left\{Q_t\right\}^{-1} Q_t diag\left\{Q_t\right\}^{-1} \tag{5}$$

Among them, $\overline{Q}$ is the unconditional correlation coefficient matrix of $\varepsilon$ ; $\circ$ is the Hadamard product.

Next, in order to characterize the "leverage effect", this article assumes that the actuals sequence of the period follows a normal distribution, and selects the tGARCH (1, 1) model to extract its standard deviation. The model equation is as follows:

$$s_t = \varphi_{0,s} + \varepsilon_{s,t} \tag{6}$$

$$f_t = \varphi_{0,f} + \varepsilon_{f,t} \tag{7}$$

$$\sigma_{s,t}^2 = c_s + \alpha_s \sigma_{s,t-1}^2 + \beta_s \varepsilon_{s,t-1}^2 + \gamma_s k_{s,t-1} \varepsilon_{s,t-1}^2 \tag{8}$$

$$\sigma_{f,t}^2 = c_f + \alpha_f \sigma_{f,t-1}^2 + \beta_f \varepsilon_{f,t-1}^2 + \gamma_f k_{f,t-1} \varepsilon_{f,t-1}^2 \tag{9}$$

Among them, $s_t$ and $f_t$ are the return rates of actuals and futures after taking the logarithm of period *t*, respectively. Let i=s,f, then *S* represents the sequence of actuals logarithmic returns, and *f* represents the sequence of futures logarithmic returns. $\varphi_{0,i}$ is the intercept term of the *i* return sequence in the mean value equation. $\varphi_{1,i}$ represents the influence of the *i* return sequence in the past period on the current *i* return sequence, and $\varepsilon_{i,t}$ represents the disturbance term of the *i* return sequence at time *t*. $\sigma_{i,t}^2$ is the conditional variance of the *i* return sequence at time *t*. $\alpha_i$, $\beta_i$, and $\gamma_i$ are the parameters to be estimated, and whether $\alpha_i + \beta_i$ +0.5 $\gamma_i$ is close to 1 reflects whether the fluctuation of *i* return sequence continues. Among them, $\gamma_i$ is used to reflect the influence of the negative and positive news of $t-1$ period on the volatility of the *i* return sequence in the current period. When $\gamma_i$ >0, it is said that there is a "leverage effect" in the *i* return sequence volatility. $k_{i,t-1}$ is a nominal variable. When $\varepsilon_{i,t}$ <0, $k_{i,t-1}$ is equal to 1; when $\varepsilon_{i,t}$ >0, $k_{i,t-1}$ is equal to 0.

### 3.2 Hedge ratio based on minimizing variance

Johanson (1960) proposed the lowest risk hedging method for the first time, and most of the subsequent studies are based on this method. When the futures ratio of h minimizes the variance of the asset portfolio, that is, when the risk is the smallest, h is the optimal hedge ratio, and the calculation formula for h is as follows:

$$h = \frac{\operatorname{cov}(\Delta S_t, \Delta F_t)}{Var(\Delta S_t)} = \rho_{s,f} \frac{\sigma_{s,t}}{\sigma_{f,t}} \tag{10}$$

Among them, $\rho_{s,f}$ is the correlation coefficient of the futures actuals logarithmic return sequence, this article uses dynamic conditional correlation coefficient to replace. $\sigma_{s,t}$ and $\sigma_{f,t}$ are the standard deviations of the actuals logarithmic return rate and the futures logarithmic return rate, respectively, obtained by the tGARCH(1,1) model.

### 3.3 Evaluation of hedging performance

Unlike arbitrage, most of the target groups for hedging are risk aversions, whose intention is to minimize risk. According to previous research, this article chooses variance to measure the magnitude of volatility, that is, the magnitude of risk, and then measures the effect of hedging by calculating the proportion of the variance reduction of assets after hedging. Let the variance reduction ratio be HE, the formula is as follows:

$$HE = \frac{VAR(U_t) - VAR(H_t)}{VAR(U_t)} \tag{11}$$

Among them, $U_t$ is the return rate of the asset portfolio without hedging, and $H_t$ is the return rate of the asset after hedging.

## 4. Empirical analysis

### 4.1 Data sources and descriptive statistical analysis

This article selects the daily closing price of INE crude oil futures (i.e. China's crude oil futures) and the daily settlement price of Daqing crude oil actuals from March 26, 2018 to June 9, 2021. Both futures actuals prices come from the Choice financial terminal. In the data processing, the data that does not correspond to the futures and actuals trading days are eliminated, and finally 755 sets of data are obtained. In order to maintain the continuity of futures data for empirical analysis, this paper selects the main continuous contract of crude oil futures of the Shanghai Energy Exchange as the daily closing price of China's crude oil futures. At the same time, in order to prevent the phenomenon of "false regression", this paper takes the natural logarithm of the two variable sequences respectively. Finally, this article counts and analyzes the various indicators of the selected data (see Table 1 below).

**Table 1. Descriptive statistics of China's crude oil futures and actuals price series**

| | Mean standard | Deviation | Skewness | Kurtosis | J-B statistic | P value |
|---|---|---|---|---|---|---|
| Actuals | 3.9417 | 0.3027 | -1.5471 | 5.5996 | 513.7852 | 0.0000 |
| Futures | 5.9739 | 0.2379 | -0.7623 | 2.3496 | 86.4342 | 0.0000 |

As shown in Table 1, the standard deviation of the actuals series is 0.3027, which is slightly higher than the standard deviation of 0.2379 of the futures series, indicating the degree of deviation of the actuals logarithmic price series from the mean, that is, volatility, which is greater than that of futures; from the perspective of skewness, The actuals sequence is -1.5471 and the futures sequence is -0.7623, both of which are lower than 0, showing a negative bias. The degree of negative bias of the actuals sequence is greater than that of the futures; from the kurtosis point of view, the kurtosis of the actuals logarithmic price sequence is 5.5996 higher than 3, indicating Its kurtosis is relatively high; from the P value of the JB test, the logarithmic price series of INE crude oil futures and Daqing crude oil actuals are significantly not normally distributed. In addition, in order to more clearly examine the correlation between China's crude oil futures and spot, this article draws two trend charts of logarithmic price series (see Figure 1 below)

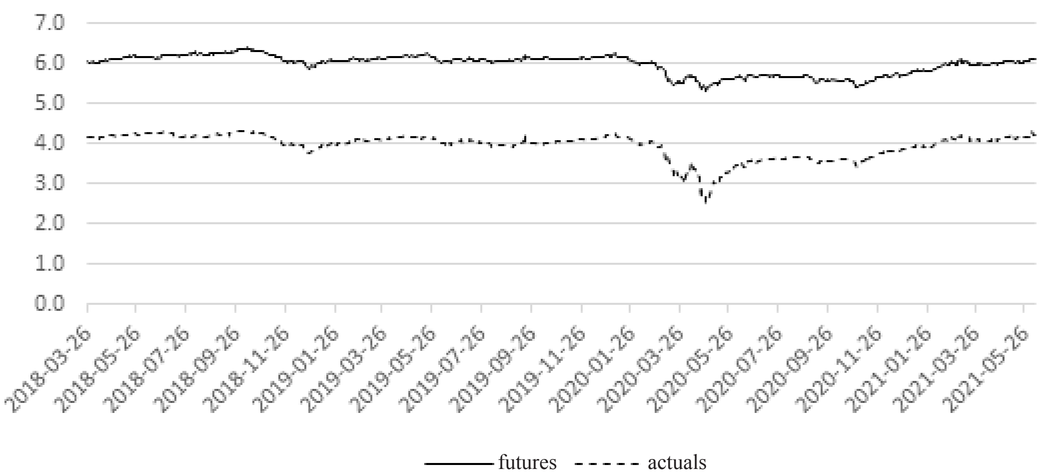


**Figure 1. The logarithmic price trend chart of futures and actuals**

As shown in Figure 1, China's crude oil futures and Daqing crude oil actuals logarithmic prices have roughly the same increase at each time node, which shows that China's crude oil futures and actuals prices have a high degree of correlation and have a basis for hedging. Secondly, the distribution range of the basis difference between the two series of futures and actuals log prices is [-2.8699, -1.7728], both of which are less than zero.

## 4.2 Stationarity test

Before modeling the time series, a stationarity test is required. This article first chooses the ADF method to test the stationarity of China's crude oil futures and actuals logarithmic price series (see Table 2 below).

**Table 2. Stationarity test results**

| | P value (price) | P value (rate of return) |
|---|---|---|
| Actuals | 0.84 | 0.0000 |
| Futures | 0.922 | 0.0000 |

Note: The original hypothesis is that the series is not stationary.

As shown in Table 2, the logarithmic price series of China's crude oil futures and actuals prices are non-stationary series at the significance levels of 10%, 5%, and 1%. Therefore, the first-order difference is taken for the futures actuals logarithmic price series to obtain a sequence similar to the return rate, and then the stationarity test is performed on the futures actuals logarithmic return sequence, and the sequence is found to be stable. Therefore, the futures actuals sequence selected in this article is a first-order single-integration sequence, which can be further tested for cointegration.

## 4.3 Cointegration test

In order to test whether the futures and actuals logarithmic prices are in a long-term equilibrium, a cointegration test is required. The methods used for cointegration test in most studies are E-G two-step method and Johansen test method. Among them, when the number of variables is 2, the E-G two-step method is more convenient. Therefore, this article chooses the E-G two-step method to conduct a cointegration test on the logarithmic return of futures actuals (see Table 3 below).

**Table 3. Cointegration test results**

| | P value | Whether there is a cointegration relationship |
|---|---|---|
| The residual sequence | 0.0000 | Yes |

Note: The null hypothesis is that there is no cointegration relationship.

As shown in Table 3, the P value of the residual series is significantly less than 1%, overturning the original hypothesis, indicating that the futures and actuals log return series have a co-integration relationship at the 1% significance level.

## 4.4 ARCH effect test

The prerequisite for constructing the DCC-GARCH model is that the sequence has ARCH effect. This article chooses to use the more common LM test method to carry out the ARCH effect test (see Table 4 below).

**Table 4. ARCH effect test results**

| | chi-squared | p-value |
|---|---|---|
| Actuals | 127.95 | 0.0000 |
| Futures | 127.09 | 0.0000 |

Note: The null hypothesis is that there is no ARCH effect in the sequence.

As shown in Table 4, the P value of the logarithmic return series of futures and actuals is close to 0, less than 1%, indicating that there is an autoregressive conditional heteroscedasticity (ARCH) effect at the 1% significance level.

## 4.5 Parameter estimation

This article uses Eviews 10 and R software to estimate the parameters of the DCC-GARCH model (see Table 5 below).

**Table 5. DCC-GARCH model estimation results**

| | $c_i$ | $\alpha_i$ | $\beta_i$ | $\gamma_i$ | dcca1 | dccb1 |
|---|---|---|---|---|---|---|
| Actuals | 0.000131<br>(0.0000) | 0.538802<br>(0.0000) | 0.288082<br>(0.0000) | 0.271073<br>(0.0000) | 0.125129<br>(0.0000) | 0.873112<br>(0.0000) |
| Futures | 0.0000363<br>(0.0001) | 0.777086<br>(0.0000) | 0.090394<br>(0.0082) | 0.127925<br>(0.0013) | | |

Note: The P value estimated by the parameter is in parentheses

It can be seen from Table 5 that the P values of all parameters of the DCC-GARCH model are less than 1%, indicating that the post-uniform sequence of actuals and futures logarithms has a "leverage effect" and the model fits well. Among them, the term $\gamma_i$ is the coefficient to measure the "leverage effect". The terms of $\gamma_i$ in this article are greater than 0, indicating that both the logarithmic return series of crude oil futures and actuals in China have an inverse "leverage effect", that is, good news affects yield fluctuations. The impact is greater than the bad news. Second, the value of $\alpha_i + \beta_i + 0.5\gamma_i$ can measure the persistence of fluctuations. The closer it is to 1, the longer the fluctuation of the sequence. The value corresponding to the actuals logarithmic return sequence is 0.962421, and the value corresponding to the futures is 0.931443, both of which are very close to 1, indicating that the volatility of the futures and actuals return sequences has a generally long duration.

## 4.6 Calculation of hedge ratio

Bringing the estimated parameters into equations (4) and (5), the dynamic condition correlation coefficient can be obtained (see Figure 2 below).

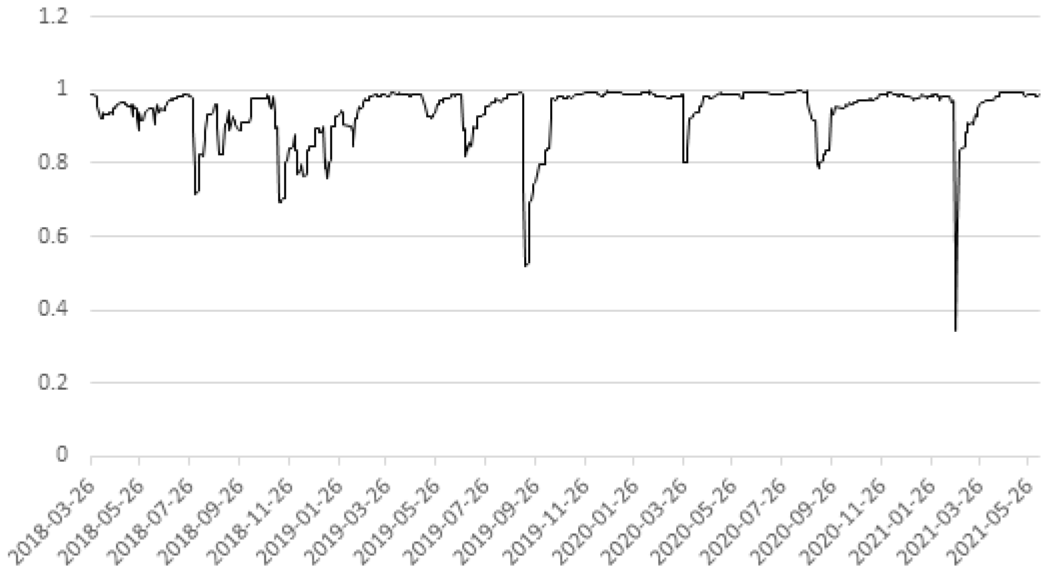


**Figure 2. The trend chart of correlation coefficient of dynamic conditions**

It can be seen from Figure 2 that the dynamic conditional correlation coefficient of China's crude oil futures and actuals is greater than 0.8 in the whole sample period, indicating that there is a high degree of correlation.

Then, substituting the dynamic conditional correlation coefficient and the standard deviation of the actuals log return sequence into equation (10), the dynamic optimal hedge ratio is obtained (see Figure 3 below).

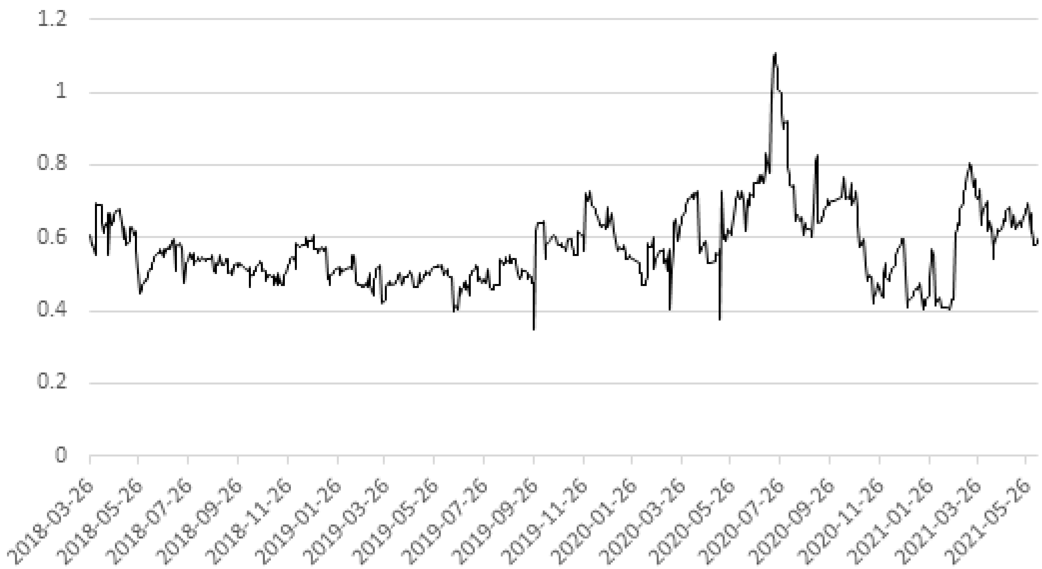


**Figure 3. Dynamic optimal hedge ratio**

It can be seen from Figure 3 that the dynamic hedge ratio obtained based on the DCC-GARCH model is generally in the range of 0.4 to 0.6, but there are significant fluctuations in the two time periods of July 2020 and March 2021.

## 4.7 Performance comparison of hedge ratio

In order to better reflect the hedging effect of the DCC-GARCH model, this article compares it with the three static hedging models of OLS, B-VAR and ECM. Since the DCC-GARCH model obtains a time-varying hedge ratio, its average value is 0.558806 and added for comparison (see Table 6 below).

**Table 6. Comparison of hedging performance of different models**

| Model | Hedge ratio | HE |
|---|---|---|
| OLS | 0.140141 | 0.716360 |
| B-VAR | 0.116950 | 0.769173 |
| ECM | 0.139806 | 0.777042 |
| DCC-GARCH | 0.558806 | 0.843660 |

It can be seen from Table 6 that the hedge ratios estimated by the three static models of OLS, B-VAR and ECM are all relatively close, in the range of 0.11 to 0.14, while the hedge ratio of the DCC-GARCH model has increased significantly. 0.558806. From the perspective of the variance reduction ratio (HE), the difference between the OLS, B-VAR and ECM models is not large. Among them, the error correction model (ECM) that takes into account the cointegration relationship between futures and stocks has the best effect, followed by B-VAR, and the OLS model with a simpler structure has the lowest HE value. The effect of the DCC-GARCH model considering the ARCH effect in terms of HE value is very significant, which is much higher than the other three models.

# 5. Conclusions and recommendations

This paper selects China's crude oil futures and Daqing crude oil spot, and establishes the DCC-GARCH model for hedging research. In addition, the optimal hedge ratio obtained from the DCC-GARCH model is compared with the OLS, B-VAR and ECM models, and the following conclusions are obtained.

(1) There is a significant reverse "asymmetric effect" between China's crude oil futures and Daqing crude oil spot. The good news will have a greater impact than the bad news, and the fluctuation of the yield series will continue for a long time.

(2) The hedge ratios obtained through OLS, OLS, B-VAR, ECM and DCC-GARCH models have performances greater than 0.7, indicating that China's crude oil futures have a better risk management function for crude oil actuals and can effectively It hedges risks.

(3) The time-varying hedge ratio obtained by the DCC-GARCH model has the best hedging performance, indicating that the dynamic hedging model is due to the static hedging model.

In response to the above conclusions, this article suggests that in the choice of hedging model, the dynamic hedging model should be preferred, and on this basis, the co-integration relationship between futures and actuals goods can be considered to improve the estimation accuracy of the model and improve the hedging performance .

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