An Empirical Research of the Consumption-based Capital Asset Pricing Model: Evidence from China's Stock Market

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Abstract: Recently, the Capital Asset Pricing Model has been widely used in the stock market. The traditional Capital Asset Pricing Model has been revised and expanded to the Consumption-based Capital Asset Model. This article doesthe research in the following ways. Firstly, this article summarizes the Capital Asset Pricing Model and empirical method. Secondly, it analyzes and processes the data worked out of the Capital Asset Pricing Model. Finally, it analyzes the empirical results.
***Keywords*:** consumption, capital asset pricing model, empirical

Introduction

In the field of finance, risk and return are always the hot topics to do the research.asset pricing is the most important contents in modern financial theory. The Capital Asset Pricing Model has also been updated and expanded with the continuous development of finance. In terms of the current development of China's capital market, the stock market has achieved certain achievements after more than 20 years development. From this perspective, this research has certain practical research value and significance.

1. Consumption-based Capital Asset Model and empirical method

In the pure exchange economy, rational expectations are more equivalent and information is more symmetrical. At this point, the utility function of capital asset model is shown in equation (1-1), as a function of the budget equation (1-2):

$$\sum_{j=0}^{\infty} \beta^{j} \mathrm{E}\left[u\left(C_{t+j}\right) \middle| I_{t}\right] \tag{1-1}$$

$$W_{\mathrm{t}+1} = \left(W_{\mathrm{t}} - C_{\mathrm{t}}\right) \sum_{i=1}^{N} \omega_{i,t} \left(1 + R_{i,t+1}\right) + Y_{t+1} \tag{1-2}$$

In the above listed formula, C_{t+j} is investor's consumption situation at the time of $t+j$; $u(\cdot)$ is the continuous function of investor's consumption of secondary strictly convex; W_t is the total wealth of the investor at time of t; Y_{t+1} is the investor's income at time of $t+1$; $\omega_{i,t}$ is theproportion of assets of i at time t in the market; N is the number of assets; β is the time discount factor, and the numerical value of β is related to the weight of consumption utility. I_t is the information obtained by investors at time of t.

In addition, there are also studies that set the $u(\cdot)$ in the utility function as a power utility function, such as formula (1-3):

$$\mathrm{u}\left(C_{\mathrm{t}}\right) = \begin{Bmatrix} \left(C_{t}^{1-\alpha}{}_{t} - 1\right)/1-\alpha, \alpha \neq 1, \alpha > 0 \\ \log\left(C_{t}\right), \alpha = 1 \end{Bmatrix} \tag{1-3}$$

Among them, the constant relative risk aversion coefficient is represented by α .

Take formula (1-1) as the objective function and formula (1-2) as constraints to form the Euler's equation for optimal conditions, such as (1-4):

$$E\left[\beta \frac{u'(C_{t+1})}{u'(C_t)} R_{i,t+1} - 1 \middle| I_t\right] = 0, i = 1, \dots N \tag{1-4}$$

In (1-4), put in the utility function of formula (1-3), we can get (1-5):

$$E\left[\beta \frac{(C_{t+1})^{-\alpha}}{C_t} R_{i,t+1} - 1 \middle| I_t\right] = 0, i = 1, \dots N \tag{1-5}$$

Normally, formula (1-5) is considered as a Capital Asset Pricing Model determined from the perspective of consumption.

In addition, to analyze the generalized method of moments theoretical model. After the generalized method of moment theoretical model was proposed in the academic field, it began to be applied to empirical research related to asset pricing. The mean value of the sample is represented by $E_T = T^{-1}\sum_{t=1}^{T}$; The number of observation periods of the sample is represented by T. Defining the error of asset pricing, such as (1-6):

$$g_T(\theta) = E_T\left[\left[M_{t+1}(\theta)R_{t+1} - 1\right] \otimes Z_t\right] \tag{1-6}$$

M_{t+1} is the random discount factor $\theta = (\alpha, \beta)'$, $R_{t+1} = (R_{1,t+1,\dots,} R_{N,t+1})'$. The instrumental variable is Z_t, and the constant term 1 is included. The Kronecker product operator is $\otimes$.

Under such model, selecting θ to minimize the pricing error and make $g_T(\theta)$ closer to zero population moment. Assuming W represents the order weighted matrix $NQ \times NQ$, the objective function of samples of generalized method of moments model can be formed as (1-7):

$$J_T(\theta) = g_T(\theta)' W g_T(\theta) \tag{1-7}$$

Among them, the estimated value of θ is represented by $\hat{\theta}$, such as (1-8):

$$\hat{\theta} = \left(\hat{D}_T' \hat{S}'^{-1} \hat{D}_T'\right)^{-1} \hat{D}_T' \hat{S}^{-1} E_T\left[1 \otimes Z_t\right] \tag{1-8}$$

In pricing model, the estimated value of error and covariance is represented by $\hat{S}_T'$ in (1-9):

$$\hat{S}_T = (1/T)\sum_{t=1}^{T}\left(\left(g_t(\hat{\theta}) - g_T(\hat{\theta})\right)\left(g_t(\hat{\theta}) - g_T(\hat{\theta})\right)'\right) \tag{1-9}$$

Among them, $g_t(\hat{\theta})$ is the estimated value of the pricing error vector at time t; $\hat{D}_T$ is the sample moment gradient, when the pricing error exceeds the parameter 0, it can be expressed as (1-10):

$$\hat{D}_T = \frac{\partial g_T(\theta)}{\partial \theta}\bigg| \theta = \hat{\theta} \tag{1-10}$$

The number of parameters to be estimated is expressed as $NQ > K$. If the number does not exceed the number of orthogonal conditions, an over-identification test needs to be carried out, such as (1-11):

$$J = NJT\left(\hat{\theta}\right) = Ng_T\left(\theta\right)' S_T^{-1} g_T\left(\theta\right) x^2\left(NQ-K\right) \tag{1-11}$$

Overall, in order to improve the reliability of the research, this research chooses a relatively simple Capital Asset Pricing Model, focusing on the (1-5) equation to explore the capital asset pricing.

2. Analyze and process the data worked out of the capital asset pricing model

2.1 Source, process and segmentation of data

China's stock market was established in 1990, and the establishment time is not too long. Moreover, because the stock market in the early time of its establishment is not very standardized, thus there are many factors that will cause fluctuations in stock returns.[1] Therefore, this study focus on the monthly data of our country's stock market from January 2010 to December 2020 for a total of 132 months. The data involved in the study are all sourced from thestatistical database of China Economic Net. After got the monthly data of the consumer price index, then adjust the consumer price index using December 2012 as the base period to obtain the relevant data of the consumer price index and consider it as an adjustment factor for variable inflation. Due to the difficulty in obtaining monthly consumption data, monthly data on total retail sales of social consumer goods can be used instead of monthly consumption data. Get the annual total population from the website of the National Bureau of Statistics.[2-3] Talking from this basis, according to the exponential growth method, the calculation of the total monthly population is realized. Divide the adjusted consumer price index by the total monthly population to get the monthly per capita consumption. $RPCR_t=C_t/C_{t-1}$ represent the growth rate of consumption. The data on the rate of return on risk assets also comes from the statistical database of China Economic Network. The adjusted consumer price index uses the logarithmic difference method to obtain the actual rate of return, respectively $R_{sh,t}$ and $R_{zh,t}$. In the Reset database, the 3-month time deposit interest rate is used as the risk-free interest rate. After the consumer price index is adjusted, the relatively risk-free interest rate is obtained, which is expressed as the formula R_{ft} . Due to the long selecting period of the sample, in order to facilitate the estimation of the changes in the stock market, the current period of samples are being divided.[4-5] Finally, the sample data from January 2010 -December 2020 is divided into four sample periods, and the total sample period is counted, making a total of five sample periods. The sample periods respectively are: January 2010-December 2011; January 2012-December 2014; January 2015- December 2018; January 2019-December 2020.

2.2 Descriptive statistical analysis of data

Table 1 shows the variable descriptive statistical analysis. From the perspective of data, the two sample periods of 2012.1-2014.12 and 2015.1-2018.12 did not exceed 1, which means that the net rate of return on risky assets is negative. In terms of the total return on risk-free assets, only thetotal return of sample period of 2010.1-2011.12 exceeds 1, which means the return on net assets is negative.

Table 1. Variable descriptive statistical analysis

Variable	2010.1-2011.12	2012.1-2014.12	2015.1-2018.12	2019.1-2020.12	2010.1-2020.12
$R_{sh,t}$	2.013(0.074)	1.984(0.055)	2.059(0.073)	1.979(0.099)	2.001(0.082)
$R_{zh,t}$	2.003(0.094)	2.989(0.065)	2.066(0.084)	1.983(0.109)	2.003(0.093)
$R_{f,t}$	2.001(0.012)	1.977(0.026)	1.894(0.032)	1.786(0.031)	1.908(0.094)
$RPCR_t$	2.009(0.078)	2.011(0.069)	2.016(0.058)	2.010(0.056)	2.011(0.065)
$\sigma_{sh,c}$	-0.001	-0.001	0.000	-0.000	-0.002
$\sigma_{zh,c}$	-0.001	-0.001	-0.000	-0.001	-0.001
$\sigma_{f,c}$	0.000	0.000	0.000	0.000	0.000

Notes: σ represents covariance of random two variables; Data in the () represents the standard error.

We found there is no significant difference between the variable and the volatility of consumption growth when we analyze the variable fluctuations. As far as the data in this study, the volatility of consumption growth in our country can explain the volatility of risky assets to a certain extent. Analysis the covariance between the analysis variables, only the covariance between 2015.1-2018.12 and consumption growthcovariance is positive, and the rest are all negative. This data shows that there is a negative correlation between the rate of return on risky assets and consumption.

3. Analysis of the empirical results of stationarity and normality tests

With the aims of avoiding the phenomenon of spurious regression, the stability of the variables is required. Therefore, the variable stationarity test shown in Table 2 is carried out. Summarizing the relevant results of the stationarity test, in terms of the risk-free asset rate of return, only the samples of 2012.1-2014.12 could not reject the unit root null hypothesis, and the variables in all other sample periods significantly rejected the null hypothesis. In other words, except that there is no risk-return ratio of risky assets in the sample period, the time series of the other periods are relatively stable.

Table 2. Variable stationarity test

Sample period	$R_{sh,t}$	$R_{zh,t}$	$R_{f,t}$	$RPCR_t$
2010.1-2011.12	-6.175***(0.000)	-6.763***(0.000)	-5.065***(-0.003)	6.976***(0.000)
2012.1-2014.12	-5.100***(0.003)	-4.863***(0.005)	-3.649(0.262)	-10.479***(0.000)
2015.1-2018.12	-5.919***(0.001)	-4.538*(0.015)	-4.831***(0.001)	-5.256***(0.002)
2019.1-2020.12	-8.973***(0.000)	-8.606***(0.000)	-4.021***(0.003)	-7.614***(0.000)
2010.1-2020.12	-7.083***(0.000)	-8.012***(0.000)	-3.084**(0.036)	-4.378***(0.013)

Note: ***, **, * indicates that the null hypothesis is significantly rejected at the levels of 1%, 5%, and 10% respectively; the value in () is the P value.

At the same time, the normality distribution test is carried out for the above mentioned variables, the results of the normality distribution test of the variables are shown in Table 3. Summarizing the results of the normality test of the variables, it is found that there are obvious differences in the results of the normality distribution of the four variables in the five sample periods. On $R_{sh,t}$, the two sample periods of 2012.1-2014.12 and 2015.1-2018.12 could not reject the null hypothesis of normality distribution, and the samples in the remaining periods significantly reject the null hypothesis of normal distribution. On $R_{f,t}$, only the samples of 2010.1-2011.12 and the samples of 2010.1-2020.12 significantly reject the null hypothesis of normality distribution, and the rest samples of the period cannot be reject it. On $RPCR_t$, the 2010.1-2020.12 sample period significantly reject the null hypothesis of normality distribution.

Table 3. Variable normality test

Sample period	$R_{sh,t}$	$R_{zh,t}$	$R_{f,t}$	$RPCR_t$
2010.1-2011.12	32.851***(0.000)	60.623***(0.000)	2.671(-0.434)	3.514(0.284)
2012.1-2014.12	1.988(0.611)	1.295(0.863)	5.306(0.116)	1.836(0.659)
2015.1-2018.12	1.478(0.789)	1.184(0.912)	3.070(0.355)	1.385(0.825)
2019.1-2020.12	6.438*(0.066)	2.651(0.438)	6.526*(0.063)	1.503(0.777)
2010.1-2020.12	22.603***(0.000)	15.246***(0.001)	18.517***(0.000)	5.634*(0.099)

Note: ***, **, * indicates that the null hypothesis is significantly rejected at the levels of 1%, 5%, and 10% respectively; the value in () is the P value.

We should pay attention that in the sample period of 2010.1-2020.12, all variables are significantly rejected in terms of obeying the null hypothesis of normality distribution, and the test results are relatively consistent. On the whole, whether the rate of return on risky assets, the rate of return on risk-free assets, consumption growth data, etc. obey the normality distribution need us to carry out a deep inspection and research. Therefore, when conducting theempirical research and test of the Consumption-based Capital Asset Pricing Model, we must choose a reasonable empirical research method according to the actual situation.

4. Conclusion

The Capital Asset Pricing Model has attracting more attention in the field of financial nowadays. From the time when the Consumption-based Capital Asset Pricing Model was put forward, scholars have always carried out empirical research on the application of the model and continue to do more expanding research of the model. On the basis of maximizing the lifetime effect of investors, using the Capital Asset Pricing Model to select assets reasonably is more in line with reality of financial field. At the same time, with the help of big data of China's stock market and empirical testing, analysis of Consumption-based Capital Asset Pricing Model can further grasp the status of adapting Capital Asset Pricing Model in China's capital market, which is also of instructive practical significance for the development of our country's stock market. This paper selects monthly data from January 2010 to December 2020, and conducts statistical and descriptive analysis of the data through Consumption-based Capital Asset Pricing Model and empirical methods. On this basis, the empirical results of stationarity and normality tests are summarized. The research results show that the pricing equation is valid in the three sample periods of 2010.1-2011.12, 2012.1-2014.12, 2015.1-2018.12, and the research results in the two sample periods of 2019.1-2020.12 and 2010.1-2020.12 are differ to the academic research results. It can be confirmed that investors are more risk-loving in the investment process, especially when the stock market is in the developing stage, investors are more likely to show a tendency to chase risk.

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