Calculation of VaR — Based on the Account Manager's Perspective

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Abstract: Value at Risk (VaR) is one of the risk measurement methods used by international financial institutions, which can be applied to stock, bond, future, option, complex derivative and other financial markets. VaR is also a primary measure for quantifying market risk and has gradually become the main basis for banks to calculate their capital requirements for market risk using internal models. This paper attempts to use three methods (variance-covariance method, historical simulation method, Monte-Carlo simulation method) to calculate VaR in both single asset and multiple assets scenarios to help account managers manage portfolios and guard against various potential risks. In addition, it compares the advantages and disadvantages of three methods in different scenarios to assist account managers adopt algorithms flexibly.

***Keywords*:** Value at Risk, variance-covariance method, historical simulation method, Monte-Carlo simulation method, A-Share market

1. Introduction

Financial risk is a major problem faced by every investor and consumer, and it is also a key issue for the survival and development of economic entities in various countries, especially financial institutions. It directly affects various activities in economic life, and also affects a country's macro decision making, especially in economic development. How to effectively control financial risks has become a topic that financial institutions and institutional investors pay more and more attention to. At present, the most popular financial risk management method is Value at Risk (VaR).

In 1996 [7], JP.Morgan presented a complete VaR research report and gave a risk metrics model for calculating VaR. Since then, VaR has become a new tool for measuring and managing market risk. With simple concept, easy to understand and convenient operation, VaR successfully quantifies various risks. It is precisely due to the many advantages of the VaR model, it has been recognized by the Basel Committee as a standard measurement tool for market risk. Generally, there are three technologies to calculate VaR. Variance-covariance method based on Markowitz's theory [5] of portfolio management, variance and covariance of a portfolio can be calculated by the mean and variance of the return rate of the underlying assets in the portfolio and the correlation between assets. Assuming that the rate of return changes of all assets in the portfolio are subject to normal distribution, and that the rate of return changes of the value of the portfolio are also subject to normal distribution. Therefore, the VaR of the portfolio can be calculated according to the probability formula and the characteristics of normal distribution.

Manganelli [4] proposed a historical simulation method for calculating VaR, and compared the estimated quantile with the real quantile to verify the effectiveness of the method. Xiao & Koenker [8] has put forward the concept of quantile regression and the interior point algorithm of quantile regression, thus quantile regression (the idea of historical simulation method) has been widely used.

The basic concept behind the Monte Carlo method [1] is to simulate a large number of random processes repeatedly running on a variable of interest (such as asset returns in the financial field), covering a wide range of possible scenarios. These variables are extracted from pre-specified probability distributions, assuming that these distributions are known, including the analysis function and its parameters. Indarwati & Kusumawati used Monte-Carlo simulation to calculate value at risk by reconstructing the revenue distribution at a location where VaR can be calculated [2].

VaR theory has been proposed for 30 years, but the use of VaR by Chinese financial institutions is not extensive, and most of them still stay at the level of relatively simple textual introductions or ALM & CAPM models. This paper starts from the perspective of account managers and uses variance covariance method, historical simulation method and Monte-Carlo simulation method to calculate VaR in single and multi assets scenarios, in order to conduct in-depth analysis from mathematical and application aspects.

2. Variance-Covariance Method

The definition of VaR is the maximum possible loss of a financial asset in a specific period of time in the future at a certain probability level (or confidence). It can be expressed as:

$$VaR_{\alpha}(V) = inf\{x \in R : P(V < -x) \leq 1-\alpha\} \tag{1}$$

where V is the amount of loss in value of a financial asset over a certain holding period, α is the given significance level, x is the absolute value of VaR. A VaR value of $|x|$ for a one daily $(1-\alpha)$ confidence means that an institution can guarantee with a probability of $(1-\alpha)\%$ that the portfolio will not lose more than $|x|$ due to market price changes within 24 hours.

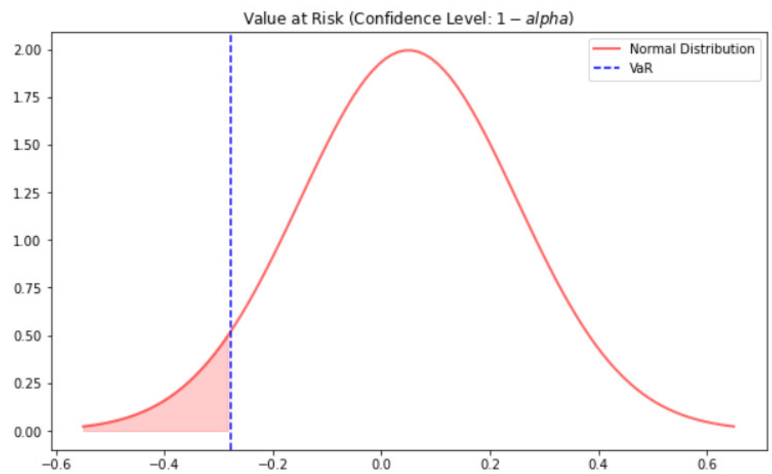

Figure 1. The Sketch of VaR

Under the normal distribution, a corresponding multiplier φ can be selected according to the confidence level, and the standard deviation of the combination is multiplied with the multiplier, then the VaR can be obtained. This method is based on estimating the standard deviation of the parameter rather than determining quantiles from the empirical distribution, thus it is called the parametric method, also called the variance-covariance method:

$$VaR = \varphi \cdot \sigma \tag{2}$$

Transform the general distribution function of return $f(R)$ into the standard normal distribution function $\Phi(\varepsilon)$, where $\varepsilon \sim N(0,1)$. The minimum value of the combination expressed by the lowest return R^* is $p^* = p_0(1+R^*)$. In general, we think of R^* as being hit by market risks thus it's negative, denotes $-R^*$. We associate R^* with skew φ of the standard normal distribution:

$$\varphi = \frac{-R^* - \mu}{\sigma} \tag{3}$$

This is equivalent to:

$$1-\alpha = \int_{-\infty}^{-R^*} f(R)\,dR = \int_{-\infty}^{\varphi} \Phi(\varepsilon)\,d\varepsilon \tag{4}$$

Consequently, the problem of VaR calculation is equivalent to finding a skew φ to make the above equation hold true. Introducing the cumulative distribution function (cdf) of standard normal distribution:

$$N(d) = \int_{-\infty}^{\varphi} \Phi(\varepsilon) d\varepsilon \tag{5}$$

The parameter φ connects the standard deviation and the quantile of the normal distribution: e.g. $\varphi = 2.33$ for $\alpha = 1\%$ and $\varphi = 1.645$ for $\alpha = 10\%$. In practice, the variance and standard deviation is estimated from historical data:

$$\sigma^2 = \frac{1}{T-1} \sum_{T}^{t=1} \left(R_t - \overline{R}\right)^2 \tag{6}$$

while R_t denotes return on period $[t-1, t]$, and $\overline{R}$ represents the average return on period $[0, t]$. Suppose an account manager holds a stock with a current value of S, annual volatility is σ, return R follows $N(0,1)$, now he wants to know what is the maximum possible loss for the next one day and the next ten days with 99% certainty. The process is:

$$F(x) = P(S_t \le x) = P\left(\frac{S_t - S}{S} \le \frac{x - S}{S}\right) = P\left(R \le \frac{x - S}{S}\right) \tag{7}$$

Standardize and combine with equation (3):

$$F(x) = P\left(\frac{R}{\sigma} \le \frac{x - S}{\sigma S}\right) = N\left(\frac{x - S}{\sigma S}\right) = 1 - \alpha \tag{8}$$

According to the properties of the inverse function we can obtain:

$$F^{-1}(1-\alpha) = x = S + \sigma S N^{-1}(1-\alpha) \tag{9}$$

The above assumes that the returns of stocks have a normal distribution with a mean of zero. The assumption of zero mean is valid for very short time periods, and for longer time measures the expression should consider calibrating the drift of asset values, assuming that the drift is μ:

$$VaR_{\text{daily}} = S(1+\mu) - F^{-1}(1-\alpha) = -S\left(\sigma N^{-1}(1-\alpha) - \mu\right) \tag{10}$$

Notice that we take VaR as positive in the real market. When calculating VaR at the same confidence level but with days $n > 1$, the expression is as follows:

$$VaR_{n-\text{days}} = VaR_{\text{daily}} \times \sqrt{n} \tag{11}$$

It should be noted here that the number of days n is determined based on the actual trading day time, not the physical time:

$$\sigma_{\text{week}} = \sigma \times \sqrt{5}; \sigma_{\text{month}} = \sigma \times \sqrt{22}; \sigma_{\text{year}} = \sigma \times \sqrt{252} \tag{12}$$

This is the case of the account manager holding a single asset, and we generalize it to the scenario of multiple assets. First we analyze a portfolio consisting of 2 assets, then calculate the variance-covariance matrix. The variance of returns for asset X can then be represented as:

$$\sigma_X^2 = \frac{1}{T-1} \sum_{T}^{t=1} \left(X_t - \overline{X}\right)^2 \tag{13}$$

In order to assess the degree of variation between assets, we compute the covariance. The covariance of asset X and asset Y returns can be represented as:

$$Cov_{XY} = \frac{1}{T-1} \sum_{T}^{t=1} \left(X_t - \overline{X}\right)\left(Y_t - \overline{Y}\right) \tag{14}$$

The correlation coefficients is:

$$\rho_{XY} = \frac{Cov_{XY}}{\sigma_X \sigma_Y} \tag{15}$$

Next, we consider a portfolio P including n assets, the weight of each asset allocation will have an impact on total assets:

$$X = \sum_{i=1}^{n} \omega_i X_i \sum_{i=1}^{n} \omega_i = 1 \tag{16}$$

The standard deviation of the portfolio P is:

$$\sigma_p = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} \omega_i \omega_j \rho_{i,j} \sigma_i \sigma_j} \tag{17}$$

where $\rho_{i,j}$ denotes the correlation coefficient between assets i and j. For the calculation of VaR, it can refer to the equation (10)(11)(12).

3. Historical Simulation Method

The non-parametric method of calculating VaR is historical simulation method, which is based on the idea that the pattern of historical returns can predict the pattern of future returns. It is directly estimated from data without the need to estimate or assume any other parameters.

To implement this methodology, the first step is to gather data on the fluctuations in market variables, such as equity prices, interest rates, and commodity prices, over an extended period of time. For instance, let's consider the daily price movements of a stock (represented by A) in the A-share market over the past year (252 trading days). This gives us 252 scenarios or cases that can serve as a reference for the future performance of A. In other words, the past 252 trading days will be indicative of what will occur tomorrow.

To establish our probability distribution for daily gains or losses, we compute the percentage change in price for A on a daily basis. Express the daily rate of returns for A as:

$$R_t = \frac{X_t - X_{t-1}}{X_{t-1}} \tag{18}$$

Afterwards, we arrange the historical returns distribution in ascending order, essentially sorting the observed returns from worst to best over the given period. Based on a chosen confidence level (probability), we can then determine the VaR for A in a one-day time horizon. If we opt for a 90% confidence level, the VaR estimate will correspond to the 10-th percentile of the probability distribution of daily returns, which is equivalent to the top 10% of the worst returns. That means there is a 90% chance that we will not incur a loss greater than our VaR estimate. The same as VaR estimate for a 99% confidence level corresponds to the top 1% of the worst returns.

4. Monte-Carlo Simulation

Assume that asset return is a stochastic process, simulate a large number of possible scenarios in the future according to stock price process. Then rank the portfolio changes under a certain scenario and provide the distribution of portfolio changes to estimate VaR values at different confidence levels. For example, the sampling times of Monte-Carlo simulation method is 100000 times, and 100000 different sample values of portfolio returns can be obtained by repeating the above steps. For a portfolio with one daily and confidence level of 95%, the value at risk corresponds to the value of the 500-th largest loss in the sample value. Modern financial market theory holds that stock price S_t obeys Geometric Brownian Motion (GBM) [6]:

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{19}$$

where μ is drift, σ is volatility, $W_t \backsim N(0,1)$ is standard normal distribution. Through Ito's lemma and Euler-Maruyama method respectively, we can obtain the explicit solution:

$$S_t = S_0 e^{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma W_t} \tag{20}$$

$$Z_{t+1} = Z_t + \left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma\sqrt{dt}\cdot\varepsilon_i \tag{21}$$

$$S_t = e^{Z_t} \tag{22}$$

where $\varepsilon_i \backsim N(0,1)$ is standard normal distribution. For simplicity, we set the volatility as a constant rather than the stochastic process of the real market. In addition, since Monte-Carlo is a random simulation, the estimated VaR values will be inconsistent each time.

5. Empirical Analysis

The account manager comes from commercial bank, then he invests the assets in the banking sector of the stock market. Suppose the account manager raises 1000000 CNY at initial time. For single asset, he invests in China Construction Bank (A-share code: 601939). For multiple assets, he invests in China Construction Bank (A-share code: 601939), China Merchants Bank (A-share code: 600036), Shanghai Pudong Development Bank (A-share code: 600000), Qilu Bank (A-share code: 601665), BANK OF QINGDAO (A-share code: 002948). The investment situation is as follows:

Table 1. Weight Distribution of the Five Stocks

Asset	CCB	CMB	SPDB	QLB	BQD
Weight	0.3	0.2	0.15	0.17	0.18

Select the close price of stocks from January 2018 to March 2024 for analysis:

Table 2. VaR for Single Asset

	1 day (95%)	1 day (99%)	10 days (95%)	10 days (99%)
Variance-Covariance	2.24×10^4	3.16×10^4	7.07×10^4	9.99×10^4
Historical Simulation	1.93×10^4	3.88×10^4	6.10×10^4	12.27×10^4
Monte-Carlo Simulation	2.21×10^4	3.10×10^4	7.00×10^4	9.80×10^4

Table 3. VaR for Multiple Assets

	1 day (95%)	1 day (99%)	10 days (95%)	10 days (99%)
Variance-Covariance	1.85×10^4	2.59×10^4	5.85×10^4	8.21×10^4
Historical Simulation	1.91×10^4	2.67×10^4	6.05×10^4	8.44×10^4
Monte-Carlo Simulation	2.36×10^4	3.25×10^4	7.46×10^4	10.28×10^4

Through the observation and comparison of Table 2 and Table 3, we can draw some conclusions:

(1) The VaR of multiple assets is lower than single asset, which shows that diversified investment can effectively reduce risk.

(2) For the same number of days, VaR with 99% confidence is higher than VaR with 95% confidence. Since the tail of the probability distribution corresponding to VaR at 99% confidence is longer over the same number of days, that means VaR at 99% confidence corresponds to a more extreme case, thus it has a higher value. However, the tail of the probability distribution corresponding to VaR with 95% confidence is relatively short, corresponding to relatively common cases, thus it has a relatively low value. As the confidence level increases, the corresponding VaR will also increase.

(3) For the VaR of single asset, the historical simulation performs well at 95% confidence, because it is easier to find regularities in the historical data of a single asset. When the confidence is low, the tail part of the probability distribution is often difficult to estimate, and the historical simulation method can estimate VaR by directly using the extreme value in the historical data, thus avoiding the error caused by making assumptions about the tail of the probability distribution. Monte-Carlo simulation perform well at 99% confidence because stochastic differential equation fits stock trends better, making it easier to identify the direction of the holding market and its associated risks.

(4) For the VaR of multiple assets, the variance-covariance method performs well at 95% confidence and 99% confidence because it makes good use of correlations between different assets, not just in weight allocation. In contrast, the Monte-Carlo simulation method performs poorly because it relies only on weights and ignores correlations in the calculation of multiple assets. Assume that there has correlation $\rho_{i(1\le i\le n-1)}$ between two Brownian motions, multiple assets have the following

correlation matrix Σ :

$$\Sigma = \begin{pmatrix} 1 & \rho_1 & \rho_2 & \cdots & \rho_{n-1} \\ \rho_1 & 1 & \rho_3 & \cdots & \vdots \\ \rho_2 & \rho_3 & 1 & & \\ \vdots & \vdots & & \ddots & \vdots \\ \rho_{n-1} & \cdots & & \cdots & 1 \end{pmatrix} \tag{23}$$

Use Cholesky decomposition [3] we can obtain matrix U (lower triangular) to participate in the calculation with ε_i :

$$UU^{*} = \begin{pmatrix} a_{11} & 0 & 0 & \ldots & 0 \\ a_{21} & a_{22} & 0 & \ldots & 0 \\ a_{31} & a_{32} & a_{33} & \ldots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \ldots & a_{nn} \end{pmatrix} \begin{pmatrix} a_{11} & a_{21} & a_{31} & \ldots & a_{n1} \\ 0 & a_{22} & a_{32} & \ldots & a_{n2} \\ 0 & 0 & a_{33} & \ldots & a_{n3} \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \ldots & a_{nn} \end{pmatrix} = \Sigma \tag{24}$$

After adjustment, the accuracy of Monte-Carlo simulation method will be improved, but we have not yet figured out how to measure the correlation between different stock prices, which may be a direction for future research.

6. Conclusion

In this paper, the mathematical analysis and empirical calculation of VaR are carried out through three methods, aiming at providing risk advisory and risk management functions for account managers' actual business. To sum up, we propose the following suggestions to account managers for their reference:

(1) When account manager hold a single asset, if the required level of confidence is not high, we recommend that the account manager use historical simulation to calculate VaR.

(2) When account manager hold a single asset, if the required level of confidence is high, we recommend that the account manager use Monte-Carlo simulation to calculate VaR.

(3) When account manager hold multiple assets, the confidence degree can be determined according to the actual situation (this factor does not affect the analysis in this case). We recommend that the account manager preferentially adopt variance-covariance method for analysis. Additionally, it can be combined with historical simulation to assist.

References

[1] Daan Frenkel. Introduction to monte carlo methods. J. von Neumann Inst. Comput., 23, 01, 2004.

[2] Ervin Indarwati and Rosita Kusumawati. Estimation of the portfolio risk from conditional value at risk using monte carlo simulation. Jurnal Matematika, Statistika dan Komputasi, 17:370–380, 05, 2021.

[3] Xiaoning Kang, Zhiyang Zhang, and Xinwei Deng. Covariance Estimation via the Modified Cholesky Decomposition, pages 887–900. 04, 2023.

[4] Simone Manganelli and Robert F. Engle. Value at risk models in finance. New York University Stern School of Business Research Paper Series, 2001.

[5] Harry Markowitz. Portfolio selection. The Journal of Finance, 7(1):77–91, 1952.

[6] Robert Merton. Lifetime portfolio selection under uncertainty: The continuous-time case. The Review of Economics and Statistics, 51:247–57, 02, 1969.

[7] J Morgan/Reuters. RiskMetrics-Technical Document. 01, 1996.

[8] Zhijie Xiao and Roger Koenker. Conditional quantile estimation for garch models (preliminary). 04, 2008.