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An Investigation into the Challenges and Strategies of Secondary Students' Geometric Thought Processes through the Lens of Van Hiele's Theory

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Abstract: This paper delves into the intricacies of the Van Hiele Theory, which posits a developmental framework for understanding how students think geometrically. The study examines the challenges secondary students face in advancing their geometric cognitive levels, particularly in transitioning from informal to formal deduction. It highlights the prevalence of students at Level 2 (Informal Deduction) and identifies a significant gap in their ability to progress to Level 3 (Formal Deduction). The paper proposes targeted strategies for facilitating this critical leap, emphasizing the need for rigorous natural reasoning, symbolization of geometric reasoning, a deep understanding of logical thinking components, and optimization of the geometric cognitive structure. Through these strategies, the paper aims to equip educators with the tools necessary to enhance students' geometric thinking and foster a robust understanding of mathematical proofs. The findings underscore the importance of a structured approach to geometric instruction that aligns with the cognitive stages of students, ultimately aiming to improve students' logical reasoning capabilities and success in mathematics.

Keywords: Van Hiele’s Theory, elementary geometry, cognitive dilemmas, pedagogical strategies

Introduction

Since the Van Hieles introduced this theory, it has been continuously developed and refined. The model most widely accepted and influential in the field, as proposed by Burger and Shaughnessy, delineates the following stages of geometric thinking: Geometry is an integral part of mathematics education, which not only requires students to master specific knowledge of shapes and spatial relationships but also demands the development of their abstract thinking and logical reasoning abilities.[1] As educators deepen their understanding of mathematics education, an increasing number of studies are beginning to focus on how students form and apply geometric thinking. Van Hiele's theory, as an important framework for understanding the development of students' geometric thinking, provides us with a systematic approach to analyze and guide students through the transition from intuitive perception to formal reasoning.

This study aims to explore the challenges that secondary school students encounter in the development of geometric thinking and to propose corresponding teaching strategies based on Van Hiele's theory.

1. Van Hiele Theory

1.1 Detailed content

Since the Van Hieles introduced this theory, it has been continuously developed and refined. The model most widely accepted and influential in the field, as proposed by Burger and Shaughnessy, delineates the following stages of geometric thinking[2]:

Level 0-Visual: At this stage, students can identify shapes by their overall and manipulate shapes like squares and triangles, as well as geometric elements such as lines and angles, based on their appearance. They are capable of drawing and imitating drawings but cannot analyze shapes using their characteristics or engage in comprehensive analysis and discussion.

Level 1-Analysis: Students at this level can identify and generalize the properties of specific shapes to solve problems. However, they are unable to explain the interrelationships between shape properties, define categories of shapes, or discern relationships between different shapes.

Level 2-Informal Deduction: Here, students can correlate shapes with their properties, formulate informal deductions, and understand the elements that constitute shapes. They explore the intrinsic attributes of specific shapes and the potential relationships between various shapes. They begin to classify shapes and use formulas, definitions, or learned properties to make informal deductive inferences.

Level 3-Formal Deduction: Students at this level appreciate the significance of mathematical proof and can demonstrate solutions to geometric problems through abstract reasoning, not just by rote memorization of formulas. They distinguish between definitions, theorems, and axioms and recognize that formal logical deduction is the foundation for theorem establishment.

Level 4-Rigor: This level is the least developed and least researched within Van Hiele's theory. At this advanced stage, students can rigorously derive theorems within various axiomatic systems and use this capability to analyze differences across geometric systems.

The Van Hieles have proposed a structured instructional sequence consisting of five distinct stages to guide this cognitive development: Pre-Instructional，Guided Orientation，Explication，Free Orientation and Integration.[3]

1.2 Challenges in secondary students' geometric thinking

Research by Wang Hongbing on the geometric cognitive abilities of first-year high school students in Nanjing revealed that the predominant cognitive level was Level 2, followed closely by Level 1, with Level 3 being the least represented.[3] The study also highlighted issues such as instability in higher cognitive levels and frequent thinking deficits. Huang Xingfeng and colleagues[4], in their investigation of geometric thinking development among children aged 7-9 in Southern Jiangsu, found that a minority had reached Level 3, while the majority of children under 7 were distributed across Levels 1 to 3. A notable improvement in cognitive levels was observed after ninth grade, with over half of the students advancing to Level 3. Nevertheless, a significant number of students remained unable to progress from Level 2 to Level 3.

As indicated by current assessments, the majority of students are at Level 2, with some at Level 1 and a very small number at Level 3. However, their ability to engage in higher-level thinking is not consistently sustained, and gaps exist in their reasoning processes. According to Van Hiele, the progression of geometric cognitive levels is sequential[5], and students cannot skip stages in their development. In light of this analysis, the paper will propose targeted strategies for facilitating the transition from the analytical stage to the informal deductive stage and from the informal deductive stage to the formal deductive stage.

2. Strategies for advancing from the analytical to the informal deduction stage

2.1 Holistic representation of geometric figures

Representation refers to the mental depiction and reflection of specific objects. A robust visual representation is the cornerstone for guiding children's cognitive development from sensory perception to rational understanding. Achieving a holistic representation of geometric figures involves a two-way observation and analysis between the figure as a whole and its parts, discerning the characteristics to richly and fully manifest them in the mind.

2.2 Elaboration of geometric elements

Elements are the fundamental constituents that make up an object. For instance, when studying triangles, a thorough analysis of their composition is required. It is essential to understand that triangles are composed of sides and angles and to examine the relationships among these elements, such as how sides relate to angles, the correlation between side lengths and corresponding angle sizes, or the properties of interior and exterior angle sums. By deeply analyzing the individual elements of a triangle, students can grasp the interplay between the elements and their collective influence.

2.3 Enhancing figure interconnectivity

Geometric shapes often exhibit both inherent and relational connections, with the latter often distinguished by definitional differences. Defining an object can take many forms, but it must adhere to four principles: the definition must be appropriate, non-circular, concise, and typically not in a negative form. For example, in elementary education, students learn about squares, rectangles, trapezoids, and quadrilaterals, which may not inherently form a connected set of concepts.

3. Strategies for advancing from informal deduction to formal deduction

3.1 Rigorization of natural reasoning

Research by Liu Jingli reveals a significant correlation between Van Hiele's levels of geometric thinking and students' mathematical reasoning abilities.[6] Consequently, the development of mathematical reasoning skills should be a focal point in the training of logical thinking. As students' thinking evolves from the informal to the formal deduction stage, they may engage in natural reasoning, which, however, lacks rigorous logic and precision. At this stage, training should aim to enhance the precision of students' mathematical reasoning and encourage the use of exact geometric language to articulate the reasoning process. According to Van Hiele's theory, the level of students' mathematical thinking development is intricately linked to the geometric language employed by teachers.

3.2 Symbolization of geometric reasoning

Symbolization is a fundamental characteristic of mathematics, and the significance of geometric reasoning symbolization lies in the necessity for the mathematical logical reasoning process to be symbolic. Geometry encompasses not only the study of concepts and propositions but also the language, reasoning processes, and methods of proof. Geometric symbols, language, and concepts form the basis for abstract thinking, and mastering them is crucial for developing "three abilities" within geometry. Thus, in the learning and teaching of geometric reasoning and proof, achieving symbolic expression is imperative.

3.3 Understanding logical thinking and its constituent elements

Understanding involves establishing non-arbitrary, substantive connections between new information and existing knowledge within a student's cognitive structure. To comprehend mathematics, one must first grasp the modes of thinking and logical relationships it entails. Mathematical concepts, judgments, and reasoning constitute the three fundamental elements of mathematical logical thinking, with concepts being the most basic unit, analogous to cells that form the logical thinking system. Mathematical concepts reflect the essential characteristics, quantitative relationships, and spatial structures of mathematics in the human mind. Due to the high level of abstraction in mathematics, forming a correct understanding of mathematical concepts can be complex. Students must undergo a series of intricate psychological processes to deeply understand these abstract concepts.

Conflicts of interest

The author declares no conflicts of interest regarding the publication of this paper.

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