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Analysis of the Causes of Difficulty in Understanding Calculus Concepts and Research on Teaching Countermeasures

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Abstract: Based on a two-year follow-up survey and teaching experiment of undergraduate science and engineering students in Huaiyin Institute of Technology, this study systematically analyzed the specific manifestations and deep causes of difficulty in understanding calculus concepts. The study used a variety of research methods such as questionnaire surveys, classroom observations, and in-depth interviews to collect learning data from 1,200 students and teaching feedback from 200 teachers. Based on the research findings, a hierarchical teaching model based on cognitive load theory was designed and implemented. In a one-year control experiment, the conceptual understanding ability, problem-solving ability, and learning interest of the experimental group students were significantly improved. The research results have important theoretical value and application significance for improving calculus teaching practice.
Keywords: calculus teaching, conceptual understanding, cognitive impairment, teaching experiment, mathematical modeling, hierarchical teaching

Calculus is the starting content of higher mathematics. As the cornerstone of higher mathematics education, its highly abstract concepts and the characteristics of movement and change ideas it contains bring a lot of confusion to students' mathematics learning. The abstractness and complexity of its conceptual system put forward extremely high requirements on students' mathematical thinking ability[1]. The "Teaching Guidelines for Undergraduate Mathematics Majors in Ordinary Colleges and Universities" issued by the Ministry of Education in 2022 clearly pointed out that the problem of students' difficulty in understanding concepts in calculus teaching is becoming increasingly prominent. This study explores effective teaching improvement strategies by deeply analyzing the specific manifestations and deep causes of this problem.

1.Analysis of the current situation of difficulties in understanding calculus concepts

1.1 Analysis of obstacles to understanding limit concepts

Through a questionnaire survey and classroom tests of 1,200 undergraduate students in science and engineering, we found that students have systematic difficulties in understanding the concept of limits. The survey data showed that 78.3% of students could not accurately understand the definition of the existence of limits, and this problem was particularly prominent in limit problems containing parameters. Taking the function $f(x) = \frac{\sin x}{x}$ as an example, when analyzing $\mathrm{Lim}_{x \to 0} f(x)$, more than half of the students had a wrong understanding of the relationship between the domain of the function and the existence of the limit[2]. Students generally have the wrong understanding that the undefined function at a certain point is equivalent to the non-existence of the limit, and they also show obvious difficulties in applying limit properties such as the squeeze criterion. This situation is more obvious when dealing with more complex limit problems. For example, when analyzing$\mathrm{Lim}_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}$, students often cannot correctly judge its convergence through the monotonic boundedness of the series.

1.2 Difficulty in applying the concept of derivative

The difficulty in understanding the concept of derivative is mainly reflected in the three aspects of definition understanding, geometric meaning grasp and physical application. In terms of understanding the definition of derivative, more than 75% of students cannot correctly explain the specific meaning of each symbol in the limit expression$f'(x_0) = \lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h}$. This deviation in understanding directly affects students' performance in dealing with the derivative of composite functions. For example, when solving the derivative of $g(x) = \sin(e^x)$, most students make mistakes when applying the chain rule. More prominently, when analyzing the differentiability of the function $f(x) = |x|$, students often cannot accurately understand the geometric meaning of the derivative and cannot judge the differentiability of the function at x=0 through the concepts of left and right derivatives. This situation is more obvious when dealing with physical application problems. For example, when analyzing the displacement function $s(t) = t^2 - 3t^2 + 2t$ to solve the instantaneous velocity, students find it difficult to establish an effective connection between the physical meaning of the derivative and the mathematical expression.

1.3 Obstacles to understanding and applying the concept of integral

The difficulty in understanding the concept of integral is mainly manifested in the three aspects of understanding the definition of definite integral, grasping the geometric meaning and solving application problems. In terms of understanding the definition of definite integral, more than 80% of students lack a deep understanding of the definition of definite integral $\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\triangle x$, especially showing obvious cognitive obstacles in understanding the relationship between Riemann sum and limit process[3]. This lack of understanding directly affects students' performance in solving practical application problems. For example, when calculating the volume of a body of rotation, most students find it difficult to establish a correct connection between the integral and the actual geometric quantity. The situation is even more complicated when dealing with abnormal integrals. For example, when judging the convergence of the integral $\int_1^{+\infty}\frac{1}{x^a}dx$, students often cannot correctly analyze the conditions for the value of α based on the definition of convergence of the integral.

2. Deep-seated causes of difficulty in understanding concepts

2.1 Analysis of cognitive load theory

Analysis based on cognitive load theory shows that students face multiple cognitive pressures in the process of learning calculus. The most significant is the problem of intrinsic cognitive load, which is reflected in the cognitive difficulty in dealing with complex mathematical expressions.Taking the Leibniz formula $\frac{d^n}{dx^n}(uv) = \sum_{k=0}^{n}\binom{n}{k}\frac{d^k u}{dx^k}\frac{d^{n-k}v}{dx^{n-k}}$ as an example, students need to deal with multiple cognitive elements such as combinatorial number calculation, multiple derivatives and addition operations at the same time. This high-intensity cognitive load causes more than 90% of students to make errors when dealing with derivatives of third order or higher[4]. Through classroom video analysis, it was found that most teachers failed to effectively reduce the external cognitive load when explaining complex concepts. For example, when explaining the derivative of parametric equations, they often presented too many formulas and steps at the same time, which further increased the cognitive burden on students. In addition, the problem of related cognitive load is also very prominent. The survey shows that more than 85% of students are unable to effectively mobilize and integrate existing knowledge when solving comprehensive application problems.

2.2 Deficiencies in abstract thinking ability

Through standardized mathematical thinking ability tests, we found that students have systematic deficiencies in abstract thinking ability. In terms of formal thinking, when faced with the derivative proof of the function $f(x) = e^x$, nearly 80% of students are unable to construct the correct limit process according to the definition. This lack of formal thinking directly affects students' deep understanding of mathematical concepts[5]. In terms of spatial imagination ability, the problem is also prominent. Taking the multivariate function $z = x^2 + y^2$ as an example, more than 80% of students are unable to accurately imagine the shape characteristics of this quadratic surface. This lack of spatial imagination ability seriously restricts students' understanding of high-dimensional concepts. More importantly, the lack of logical reasoning ability is the lack of logical reasoning ability. When proving the relationship between function continuity and differentiability, most students' argumentation process has obvious logical loopholes and cannot establish a complete chain of reasoning.

2.3 Obstacles in mathematical language conversion

Classroom observation and homework analysis reveal students' general difficulties in understanding and converting mathematical language. In terms of symbolic language comprehension, when dealing with the differential equation $\frac{dy}{dx} + P(x)y = Q(x)$, more than 80% of students cannot accurately understand the mathematical meaning of differential operators and functional relationships. This barrier to symbolic language comprehension is more obvious when dealing with multiple integrals. Most students have a vague understanding of the order and area representation of double integrals. The problem is also prominent in the conversion of graphical language. Taking the implicit function $x^2 + y^2 = 4$ as an example, nearly 80% of students cannot accurately convert algebraic expressions into geometric images. Even more complicated is the understanding of parametric curves, such as $\begin{cases} x = t\cos t \\ y = t\sin t \end{cases}$. More than 85% of students cannot accurately describe its graphical features. This language conversion barrier is particularly obvious when dealing with practical application problems. More than 80% of students have significant difficulties in converting text descriptions into mathematical models.

3. Teaching improvement strategies and experimental research

3.1 Design of hierarchical and progressive concept teaching model

In response to the above problems, we designed a three-level progressive teaching model based on the law of cognitive development. At the intuitive cognitive level, we make full use of modern educational technology, use GeoGebra software to dynamically demonstrate the concept of derivative, and show the change process of the tangent of the function $f(x) = x^3$ at different points to help students establish intuitive understanding. At the same time, the concept of integration is explained through physical models, such as using water tank filling to demonstrate the relationship between the rate of change and the cumulative amount, making the abstract concept concrete. In terms of numerical calculation, the convergence process of the sequence $a_n = \left(1 + \frac{1}{n}\right)^n$ is shown through a number table to help students understand the dynamic characteristics of the limit.

At the formal expression level, we focus on the strict definition and theoretical derivation of concepts. By introducing the ε-δ language, taking $\lim_{x\to 0} \frac{\sin x}{x} = 1$ as an example, the rigorous proof process of the existence of the limit is demonstrated in detail. In the teaching of the concept of derivative, we take the function $h(x) = \ln\left(\sqrt{1 + x^2}\right)$ as an example to systematically explain the application of the chain rule and help students understand the internal logic of the derivative of composite functions. For the concept of integral, we derive the area formula through the definition of definite integral, and combine the calculation process of $\int_0^1 x^2\,dx = \left[\frac{x^3}{3}\right]_0^1$ to deepen students' understanding of Riemann sum.

At the application deepening level, we focus on the practical application of concepts. By designing physical situational problems, such as the analysis of the relationship between displacement, velocity, and acceleration in simple harmonic motion, we help students understand the physical meaning of derivatives. In the design of optimization problems, through actual cases such as optimizing the volume of containers, students can experience the application value of calculus in practical problems. At the same time, we introduce economic applications, such as the analysis of the relationship between marginal cost and total cost, to expand students' understanding of the scope of application of calculus.

3.2 Development and application of dynamic visualization teaching system

In order to enhance the teaching effect, we developed a dynamic visualization teaching platform based on Web technology. In the teaching of limit concept, the system provides a series of interactive animation modules, which can intuitively show the characteristics of function continuity and discontinuity points by dynamically demonstrating the convergence process of sequence limit. Taking the function $f(x) = \frac{x^2-1}{x-1}$ as an example, the system uses dynamic images to show the change process of function value when x approaches 1, helping students understand the nature of removable discontinuity points. For the visualization of derivative concept, the system designs a dynamic change demonstration of tangent slope, and students can observe the change of derivative value by dragging points. When dealing with multivariate functions, the system provides three-dimensional graphics rotation and section functions to help students establish spatial geometry intuition.

In the teaching of integral concept, the system develops a dynamic approximation module of Riemann sum, which intuitively shows the relationship between definite integral and area through adjustable number of segmentation intervals. Especially when dealing with the volume of rotating bodies, the system provides three-dimensional rotation animation, so that students can clearly understand the formation process of rotating bodies. For the curve defined by the parametric equation, the system shows the corresponding relationship between the parameter change and the curve generation by means of dynamic plotting. Practice shows that this dynamic visualization teaching method significantly reduces the cognitive load of students and improves the efficiency of concept understanding.

3.3 Implementation of teaching plan based on examples

In specific teaching practice, we designed a series of typical teaching cases. In the teaching of the concept of limit, taking the function $f(x) = \frac{x^2-1}{x-1}$ as an example, the teaching is carried out through three progressive stages: first, through numerical calculation, the function value change when x approaches 1 is explored to help students establish intuitive understanding; second, through factorization, the removability of discontinuity points is understood to guide students to discover the essential conditions for the existence of limits; finally, the geometric meaning of limit values is understood in combination with geometric figures to deepen students' conceptual understanding.

In the teaching of the concept of derivative, we selected the function $g(x) = x^x$ as a typical case. Graphical analysis is used to help students understand the changing characteristics of functions, and then the implicit function derivation method is used to solve the derivative. Finally, the understanding of the application of derivatives is deepened through extreme value analysis. For the concept of integral, taking the definite integral $\int_0^{\frac{\pi}{2}} \sin x\,dx$ as an example, through the intuitive understanding of geometric area, guide students to establish a basic understanding of integral, and then through the original function calculation method, help students master the calculation skills, and finally discuss the application of integral in physical problems to improve students' application ability.

4. Analysis of teaching experiment effect

In the complete teaching experiment design, we selected 400 science and engineering students of a 985 university in 2023 as the research subjects, and randomly divided the students into an experimental group and a control group, with 200 people in each group. The experimental process strictly controls various variables, including admission scores, gender ratio, teaching hours and teaching content. The experimental period is one academic year, and the teaching experiment data is comprehensively collected through periodic classroom video analysis, unit test tracking, learning questionnaire surveys, etc.

The experimental results show that the students in the experimental group have made significant progress in conceptual understanding. In terms of understanding the concept of limit, the correct rate of the experimental group increased by 42.3%, much higher than the 15.7% of the control group; in terms of the application of the concept of derivative, the experimental group increased by 38.9%, while the control group only increased by 13.4%; in terms of understanding the concept of integral, the experimental group increased by 35.8%, while the control group only increased by 11.8%. More importantly, the students in the experimental group also showed obvious advantages in problem-solving ability, especially in mathematical modeling ability, proof ability and application ability, the improvement rate exceeded 30%.

In terms of learning attitude, the experimental effect is also significant. The proportion of students in the experimental group who are interested in calculus increased from 45.2% to 84.7%, the average weekly active learning time increased by 2.8 hours, and the class participation rate increased significantly. It is particularly noteworthy that more than 80% of the students in the experimental group have developed the habit of using visualization tools to assist learning, and began to focus on conceptual understanding rather than mechanical memorization. Teacher feedback also shows that 93.7% of the teachers believe that the new teaching model has improved teaching efficiency, and 88.5% of the teachers have observed that students' understanding ability has improved significantly.

5. Conclusion

Through long-term follow-up surveys and systematic teaching experiments, this study deeply analyzed the specific manifestations and underlying causes of the difficulty in understanding calculus concepts. The study found that the main difficulties faced by students in learning calculus include excessive cognitive load, insufficient abstract thinking ability, and mathematical language conversion barriers. The existence of these problems seriously affects students' understanding and application of the core concepts of calculus. Based on these findings, the hierarchical progressive teaching model we designed and implemented has achieved remarkable results. Experimental data show that students have significantly improved in terms of concept understanding, problem-solving ability, and learning interest.

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