Region - Water Conservancy, 2026, Volume9, Issue2
DOI: 10.32629/rwc.v9i2.5392
ISSN Online: 2630-4902

Hydraulic simulation of pumped water source systems through well fields: Part 2

Yaset Martínez Valdés, Félix Abelis Riaño Valle

Department of Hydraulic Engineering, Center for Hydraulic Research (CIH), Faculty of Civil Engineering, Technological University of Havana "José Antonio Echeverría" (CUJAE), Havana

Abstract: In this second part on the topic of hydraulic simulation of pumped water supply systems, focusing on well fields, the virtual pump method is addressed. This methodology is presented as a simple procedure for solving problems involving pumps connected in parallel when the installation conditions and/or hydraulic characteristics of the coupled pumps are not identical. It should be understood that virtual rotodynamic pumps are imaginary machines that lack secondary suction and discharge pipes. They consist of the real pump and all the fittings and pipes up to the connection point (node) with the main discharge pipe. These pumps are characterized by always operating against the same total head when running in parallel. The study recalculates the engineering example presented in Part 1 of this series, comparing the results derived from the virtual pump method against those obtained via the nodal iteration method covered in the first paper.

Key words: simulation; hydraulics; source systems; pumping; field wells

1 Introduction

The hydraulic simulation of pumped source systems in steady state consists of obtaining the flow rates circulating through the pipes and the head at the nodes of the system using the system equilibrium equations (mass conservation equation, or continuity equation and the energy conservation equation) from knowing: point consumption at the nodes (if any), the piezometric head at least one node, and the relevant characteristics of the pipes (diameter, roughness and length) and the rest of the system elements (pumps, valves, accessories, etc.) (Cabrera 2009).

The simulation process allows for the evaluation of the responses of pumped source systems under specific operating conditions. Several methods exist for simulating pumped source systems, including well fields (multinodal interconnected branched systems). These methods can be grouped into several categories: Newton-Raphson-based methods; Hardy-Cross-based methods; linear-based methods; methods based on topological network formulations (Gradient Method); node iteration methods; and the Virtual Pump method, among others (Fuertes et al., 2002) (Martínez and Riaño, 2025).

In the first part of a series of articles on this topic, the node iteration method was presented, applied to a common practical example in engineering practice (Martínez and Riaño 2025). In this second part, the objective of this work is to develop the last of the aforementioned methods, the Virtual Pump method. This is a simple and practical calculation procedure, recommended for cases involving rotodynamic pumps connected in parallel, when the installation conditions and/or hydraulic characteristics of the coupled pumps are not the same (Case 4, general case) (Martínez and Riaño 2010).

2 Development

The Virtual Pump method, although simpler than the iterative methods mentioned above, has been little studied and applied. The concept of virtual pumps simplifies the hydraulic calculation of the actual operating points and, in general, the physical understanding of the problem. If, for a given pumping system, the characteristic curve of the virtual pump includes both the suction and discharge pipes up to the discharge point, the length of the system against which pumping would occur would be zero, and the corresponding head losses would be negligible. Consequently, the characteristic curve of this system will be given simply by the values of static head versus flow rate (Galvis and Castilla 1993).

Rotodynamic pumps operating in parallel influence each other: the flow rate, head, power, and efficiency of each depend essentially on the individual load regimes of the pumps operating together. The operating cases for rotodynamic pumps in parallel are (Pardo and Ruiz 1980, Martínez 2011):

- Case 1: Pumps with identical hydraulic characteristics, operating against identical piping systems up to the point where the common discharge pipe begins (node). In this case, the previously mentioned conditions are met; that is, the pumps will be operating against the same head, therefore, the principle of adding the flow rates for equal heads will be fully satisfied.
- Case 2: Pumps with identical hydraulic characteristics, operating against different piping systems up to the point where the common discharge pipe begins (node). In this particular case, since the piping systems are different up to the point where the secondary discharge pipes of the pumps join, it can be stated that these pumps are not operating against the same head, as they are, ultimately, operating against different systems.
- Case 3: Pumps with different hydraulic characteristics, operating against identical piping systems up to the point where the common discharge pipe begins (node). In this scenario, the piping systems are the same for each pump, but, because the pumps are different, the flow rates will also be different. Consequently, the losses that occur in each section up to the junction point will be different, resulting in different curves for the individual systems. This implies that the pumps are operating against different heads at their outlets.
- Case 4: Pumps with different hydraulic characteristics, operating against different piping systems up to the point where the common discharge pipe begins (node). This is the so-called general case, for which it is evident, from everything previously explained in cases 2 and 3, that the pumps will be operating against different heads.

2.1 Analytical treatment of the characteristic curves of rotodynamic pumps coupled in parallel

The equations that model the three characteristic design curves of rotodynamic pumps coupled in parallel for the case with equal hydraulic characteristics are (Turiño 1996, Martínez 2011):

Load-capacity curve (parallel), $(H_P\text{-}Q)$: $H_P=A\pm\frac{B}{n_b}Q-\frac{C}{n_b^2}Q^2$ *(1)*

where: HP: head developed by the pumps coupled in parallel, (m); *A*: coefficient of the polynomial representing the *H-Q* curve that defines the value of the head developed by the pump for zero flow or closed valve, (m); *B* and *C*: coefficients of the polynomial representing the *H-Q* curve that are a function of the head losses inside the pump, (s/m^2), (s^2/m^5) respectively; Q: flow rate driven by the combination of the pumps in parallel, (m^3/s) and n_b: number of pumps working in parallel.

Power-capacity curve (parallel), $(P_P\text{-}Q)$: $P_P=Dn_b\pm EQ\mp\frac{F}{n_b}Q^2$ *(2)*

where: P_P: power absorbed by the pumps working in parallel, (kW); *D*: coefficient of the polynomial representing the *P-Q* curve, which defines the value of the power consumed by the pump for zero flow, (kW) and *E* and *F*: coefficients of the polynomial representing the *P-Q* curve dependent on the power losses of the pump, ($s\cdot kW/m^3$), ($s^2\cdot kW/m^6$) respectively.

Efficiency-capacity curve (parallel), $(\eta_P\text{-}Q)$: $\eta_P=\frac{G}{n_b}Q-\frac{H}{n_b^2}Q^2$ (3)

where: η_P: efficiency of the combination of pumps in parallel, (dimm.) and G and H: coefficients of the representative polynomial of the η-Q curve, (s/m^3), (s^2/m^6) respectively.

The $NPSH_r$ behavior of pumps does not change when they are placed in series or in parallel, since by definition it is a variable that characterizes the suction capacity of a pump and is therefore independent of the type of coupling.

2.2 Virtual pump method

In everyday practice, different procedures have been used to determine the operating points of each pump, whether operating individually or in parallel, that discharge into the same main pumping system. These procedures can be summarized as follows:

1. The pressure losses in the suction and secondary discharge of each pump (excluding the main discharge pipe or system) are disregarded for the calculation of the characteristic curve of the piping system.

2. Disregard the differences in pressure losses that may exist in the secondary pumping and suction of the pumps and include the pressure losses that are considered important in this part of the installation for the calculation of the characteristic curve of the main pumping pipe system.

Since the two previous criteria assume equal losses in the portion of the piping system belonging to each machine, it follows that if the pumps operate in parallel and draw from the same suction level, they must necessarily operate against the same total head. It should be noted that calculation procedures based on the previous criteria, particularly the second, yield good results for some practical problems, especially when the head losses in each pump's own piping are small compared to those in the main discharge pipe and therefore do not significantly determine the shape of the characteristic curve of the system against which pumping occurs (Castilla and Galvis 1993).

Conceptually, virtual pumps are imaginary machines that lack secondary suction and discharge pipes. These pumps consist of the actual pump supplied by the manufacturer and all fittings and pipes up to the point of connection with the main discharge pipe. These systems are characterized by always working against the same total head when operating in parallel, without requiring assumptions that could lead to only approximate solutions, as would be the case with the previous criteria. This concept of virtual pumps greatly simplifies the hydraulic calculation of the actual operating points and, in general, the physical understanding of the problem (Martínez and Riaño 2024).

If, for a given pumping system, the characteristic curve of the virtual pump is taken to include both the suction and discharge pipes up to the discharge point, the length of the system against which pumping would occur would be zero, and the corresponding head losses would be negligible. Consequently, the characteristic curve of this system would be given simply by the head versus flow rate values (Martínez and Riaño 2025).

Technically speaking, virtual pumps are analogous in their hydraulic behavior to the installation of a diaphragm between the discharge flange of a rotodynamic pump and the discharge pipe flange. It's important to note that this constitutes a pure throttling, the losses of which directly affect the pump's head-capacity characteristic curve. As is known, the pressure losses (head losses) due to the diaphragm follow a quadratic or parabolic curve; therefore, the new pump characteristic curve, after installing the calibrated diaphragm, differs from the previous curve at all points due to the pressure drop (Martínez and Riaño 2025).

This method can be used for any of the four pump design schemes operating in parallel; however, it is especially recommended for case 4 (the general case). As mentioned, this is an alternative calculation methodology to numerical methods for simulating pumping systems, specifically for pumped source systems. Hydraulically, it is a procedure for addressing the problem of the operation of rotodynamic pumps working in parallel in branched pipelines with independent and shared pipe sections.

2.3 Introduction to the virtual pump method

Using Figure 1 as a reference, Bernoulli's equation is applied from the water levels in the suction tanks of each pump to point A (the common or central node of the pumped supply system), where there is a common head H_A (piezometric head at node A) for the connections between pumping stations B1 and B2 and node A. In this case, the flow rates of the individual pumps are added together at this node, the beginning of the common section. This implies that the total pump heads must be equal at this point (SIHI 2003).

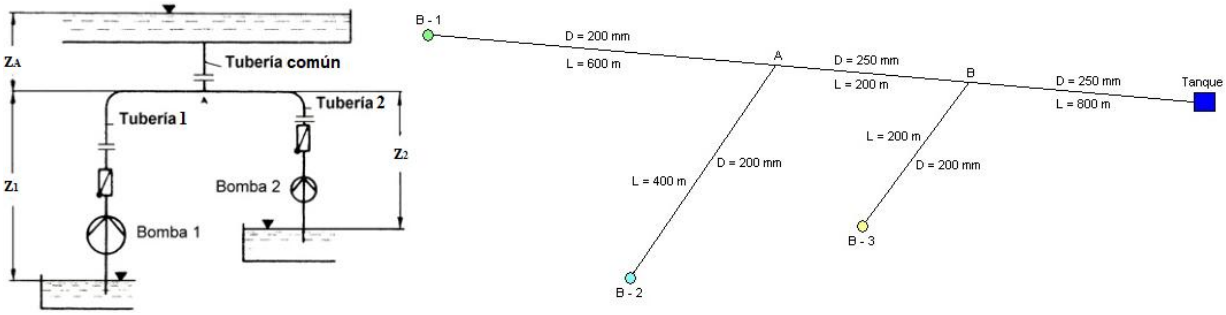

Figure 1. Operation of rotodynamic pumps with different hydraulic characteristics in parallel with independent and common pipe sections (Case 4) (left image). Example of an operating scheme of a pumped source system for a well field (right image).

The total head at node A is obtained from the H vs. Q characteristic curves of each individual pump, reducing them according to the system curves of each individual system (non-common sections). For system B1-A, we obtain:

$$H_1 \pm Z_1 = H_A + h_{fB1\text{-}A} \qquad (4)$$

While on branch B_2-A:

$$H_2 \pm Z_2 = H_A + h_{fB2\text{-}A} \qquad (5)$$

where: H_1: total head of pump 1, (m); H_2: total head of pump 2, (m); H_A: head at node A, (m); Z_{s1}: height of the water level in the suction tank of pump station 1, with respect to an established reference plane, (m); $Z_{s2:}$ height of the water level in the suction tank of pump station 2, with respect to an established reference plane, (m); $hf_{B1\text{-}A}$: head losses through branch $B_{1\text{-}A}$, (m) and $hf_{B2\text{-}A}$: head losses through line $B_2\text{-}A$ (m).

Equations 4 and 5 provide the theoretical basis for analyzing the behavior of pumps operating in parallel. In their general form, and for any number of pumps connected to the same node (node A in this example), these equations are transformed to obtain the general equation for a virtual pump B_{nA} (expression 6). This transformation process can be seen graphically in Figure 2.

$$H_A = H_n \pm Z_n - h_{fBn\text{-}A} = H_{nA} \qquad (6)$$

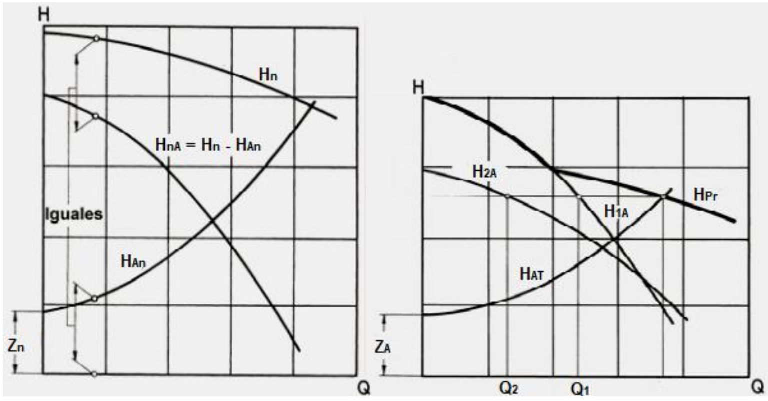

Figure 2. Reduced characteristic curve of a pump n at point A (left). Characteristic curve for the parallel combination of two virtual pumps working against a common pipe (right).

These virtual or reduced head-capacity curves can be coupled in parallel, as shown in Figure 2 (right-hand image), by summing the flow rates at the same total head. The intersection of this H_{Pr}-Q curve with the curve of the common pipe system (pipe 3 in the left-hand image of Figure 1) gives the total flow rate.

The individual flow rates for each pump are obtained by the intersection of the horizontal line (right-hand image in Figure 2) with the individual reduced characteristic curves. From these flow rates, the head (reduced and actual), power, efficiency, and $NPSH_r$ values can then be obtained for each pump.

The application of the virtual pump method in a multinodal pumping source system composed of a well field is detailed below (Martínez and Riaño 2024):

1. Determine (graphically or analytically) the actual load of the n pumps for different reference flow rates.

2. Perform dynamic correction (reduction of the actual pump head due to head losses in the individual system pipes of each pump) for each pump. This can be done graphically or analytically.

3. Calculate the static correction, which is obtained from the difference between the water levels in each suction tank and the virtual level established for the overall hydraulic system, Z_{sv}. The latter could be calculated in several ways: as an average of the water levels in all the suction tanks of the pumping system, by setting a fixed value for Z_{sv}, which could be the smallest value of all the levels, the largest, or the value chosen as the standard for the n pumps. The disadvantage of the first option is that it would require computing *n* static corrections, while for the other three options, the number of static corrections to be performed would be n-1.

However, from the standpoint of ease of calculation, the latter choice would be the best option. While, for the sake of greater efficiency in calculations and a better understanding of the conceptual idea of the procedure, it is customary to use a single value for the virtual suction level, several values can be used depending on each subsystem. In that case, the notation established for the virtual suction levels of each subsystem would be Z_{vNSn}. Taking the well field in Figure 1 as an example, we would have Z_{vNSA} (for subsystems B-1, B-2, and secondary node A) and Z_{vNSB} (for subsystem B-3, secondary node A, and main node B).

4. Calculate for each pump the virtual head corresponding to each reference flow rate according to the following expression:

$$H_{nN} = H_n - hf_n - (Z_{sv} - Z_n) \qquad (7)$$

Where: H_{nN}: virtual head of pump *n*, (m); H_n: actual head of pump n, (m); hf_n: head losses in the individual system of pump *n*, (m); Z_{sv}: piezometric level of the virtual level established for the general pumping system, (m), and Z_{sn}: water level in the suction tank (dynamic level in the well) of the pump station n, (m).

5. Obtain (graphically or analytically) the head-capacity characteristic curves of the virtual pumps B_{nN} at the node.

6. Combine (graphically or analytically) the *n* virtual pumps in parallel to obtain the equivalent virtual pump, $B_{1\text{-}n}$, at the node.

Conceptually, once this last step is carried out, there would be a virtual pumping station at the node (example: node A for figure 1 (image on the left)), where the n virtual pumps will be installed with their virtual equivalent in parallel, whose water level in the suction tank will be the virtual level established for the system, Z_{sv}.

Taking the image on the left of Figure 1 as an example, the pumping system would be simplified to a virtual pumping station-pipe-discharge tank scheme. The system could be further restructured by dynamically correcting the combination of virtual pumps in parallel, $B_{1\text{-}n}$, starting from the common pipe. In theory, this would be equivalent to having a pumping station with a virtual pump, $B_{1\text{-}n\text{-}T}$, coupled to the suction tank or reservoir (common pipe length equal to zero).

Regarding the notation of virtual pumps during the process, for practical reasons, it is advisable to follow the approach proposed by Martínez and Riaño (2024), which uses a coding system analogous to the numerical hydrological classification of rivers. In this sense, the following general notation is formulated: $H^{m}_{1\text{-}n}$; where the superscript m represents the number of virtualizations performed up to a specific node, while the subscript *1-n* represents the number and specific pumps that have participated in the virtualization process up to that point. This notation criterion is recommended for large well field systems with multiple nodes.

For a better understanding of the methodological procedure for applying the Virtual Pumps Method, it will be applied to an example of a pumped source system as illustrated in the image on the left of Figure 1.

Example application for a multinodal pumped source system (well field)

The pumped water supply system will provide continuous service 24 hours a day at a constant flow rate. Table 1 provides information on the hydraulic characteristics of the pumped water supply system, which consists of a well field as illustrated in Figure 1. The piezometric head at the discharge pipe outlet of the regulating tank is Z_t = 50.00 m. High-density polyethylene (HDPE) pipes will be used for all piping in the pumped water supply system. The absolute roughness of the HDPE material is 2.5 × 10^{-6} m. The nominal design pressure of these pipes will be 10 atm (PN 10). A recommended velocity range of 1–1.8 m/s (INRH 2006) has been used as an indicative value for the design and operation of the system. A kinematic viscosity value of 10^{-6} m²/s is assumed.

Table 1. Information on the hydraulic characteristics of the well field (See figure 1)

Flow rate (L/s)		Pipeline length (m)		Elevations and dynamic water levels (m)	
Design flow rate of the source system	250.00	Main trunk line CM (also designated main pipeline CP)	5000.00	Primary node *NP*/ central node *NC*, Z_{NP}	30.00
Design flow rate of Pump 1	60.00	Secondary feeder pipeline 1 CS_1	3000.00	Secondary node 1 NS_1, Z_{NS1}	23.00
Design flow rate of Pump 2	60.00	Secondary feeder pipeline 2 CS_2	2000,00	Secondary node 2 NS_2, Z_{NS2}	25.00
Design flow rate of secondary feeder pipeline1 CS_1	120.00	Pressure discharge pipe for secondary pipeline 1 TIS_1	1000.00	At Well 1, Z_{S1}	10.00
Design flow rate of Pump 3	70.00	Pressure discharge pipe for secondary pipeline 2 TIS_2	900.00	At Well 2, Z_{S2}	12.00
Design flow rate of Pump 4	60.00	Pressure discharge pipe for secondary pipeline 3 TIS_3	780.00	At Well 3, Z_{S3}	7.00
Design flow rate of secondary feeder pipeline 2 CS_2	130.00	Pressure discharge pipe for secondary pipeline 4 TIS_4	500.00	At Well 4, Z_{S4}	8.00

Diameters of the hydraulic system pipes (m)

D_{CM}	D_{CS1}	D_{CS2}	D_{TIS1}	D_{TIS2}	D_{TIS3}	D_{TIS4}
0.5552	0.3524	0.3966	0.2776	0.2776	0.2776	0.2776

The representative polynomials of the load-capacity characteristic curve are presented, fitted to a second-degree equation as proposed by expression (1), for the four rotodynamic pumps, with pumps 1 and 2 being equal (Q in m^3/s and H in m).

$$\text{Pumps 1 and 2: } H_{1,2} = 85.989 - 146.46Q - 4{,}087.63Q^2 \qquad (8)$$

$$\text{Pump 3: } H_3 = 88.304 - 86.643Q - 4{,}692.857Q^2 \qquad (9)$$

$$\text{Pump 4: } H_4 = 73.760 - 22.276Q - 4{,}782.313Q^2 \qquad (10)$$

Applying steps 1 through 4 described in the virtual pump methodology, the results shown in Table 2 are obtained. The notation proposed by Martínez and Riaño has been chosen to name the virtual heads during the process. The head losses will be calculated using the Darcy-Weisbach equation, with the friction coefficient, *f*, determined by the Swamee-Jain formula within its application limits.

$$hf = 0.0826 f \frac{L_{eq}}{D^5} Q^2 \qquad (11)$$

Where: *hf*: head losses in the pipeline, (m); *f*: friction coefficient of the pipe, (dimensionless.); L_{eq}: equivalent length of the system, (m); *D*: inner diameter of the pipe, (m), and Q: flow rate, (m^3/s).

$$f = \frac{0.25}{\left[log \left(\frac{\varepsilon}{3.7D} + \frac{5.74}{{N_R}^{0.9}} \right) \right]^2} \qquad (12)$$

Where: ε: absolute roughness of the pipe associated with the material, (m); *D*: inner diameter of the pipe, (m), and N_R: Reynolds number that characterizes the flow in the pipe, (dimensionless).

$$N_R = \frac{vD}{\nu} = \frac{4Q}{\pi D \nu} \qquad (13)$$

Where: N_R: Reynolds number that characterizes the flow in the pipe, (dimensionless); *v*: average flow velocity in the pipe, (m/s); *D*: inner diameter of the pipe, (m); : kinematic viscosity of the fluid, (m^2/s), and *Q*: flow rate in the pipe, (m^3/s).

Regarding the determination of the static correction for the overall hydraulic system, the first step is to define the virtual water level, Z_{sv}, that will be established for the well field. Conceptually, as mentioned above, a virtual suction head could be set for each subsystem, but it is more convenient, based on the calculations, to use a single value for the overall hydraulic system. Of the possible options, it is proposed that this be the average of the dynamic water levels in the four wells, that is, 9.250 m. Therefore, the static corrections would be calculated as follows:

($Z_{sv} - Z_1 = 9.250 - 10.000 = -0.750$ m) and ($Z_{sv} - Z_2 = 9.250 - 12.000 = -2.750$ m) for pumps 1 and 2 (subsystem 1)

($Z_{sv} - Z_3 = 9.250 - 7.000 = 2.250$ m) and ($Z_{sv} - Z_4 = 9.250 - 8.000 = 1.250$ m) for pumps 3 and 4 (subsystem 1)

Table 2. Data of the load-capacity characteristic curves of the pumps (real and their virtual)

Flow rate	Head of real pumps				Head losses in the independent piping system of each pump				Virtual pump loads (at secondary node 1)			
Q (m^3/s)	H_1 (m)	H_2 (m)	H_3 (m)	H_4 (m)	h_{fTIS1} (m)	h_{fTIS2} (m)	h_{fTIS3} (m)	h_{fTIS4} (m)	H^I_1 (m)	H^I_2 (m)	H^I_3 (m)	H^I_4 (m)
0.000	85.989	85.989	88.304	73.760	0.000	0.000	0.000	0.000	86.739	88.739	86.054	72.510
0.010	84.116	84.116	86.968	73.059	0.106	0.096	0.083	0.053	84.759	86.770	84.635	71.756
0.020	81.425	81.425	84.694	71.402	0.366	0.329	0.285	0.183	81.809	83.846	82.159	69.969
0.030	77.916	77.916	81.481	68.788	0.757	0.682	0.591	0.379	77.909	79.985	78.640	67.159
0.040	73.590	73.590	77.330	65.217	1.273	1.145	0.993	0.636	73.068	75.195	74.087	63.331
0.050	68.447	68.447	72.240	60.690	1.906	1.715	1.487	0.953	67.291	69.482	68.503	58.487
0.060	62.486	62.486	66.211	55.207	2.653	2.388	2.070	1.327	60.583	62.848	61.892	52.630
0.070	55.707	55.707	59.244	48.767	3.512	3.161	2.739	1.756	52.946	55.297	54.255	45.762
0.080	48.111	48.111	51.338	41.371	4.479	4.031	3.493	2.239	44.383	46.831	45.595	37.882
0.090	39.698	39.698	42.494	33.018	5.552	4.997	4.331	2.776	34.896	37.451	35.913	28.992
0.100	30.467	30.467	32.711	23.709	6.730	6.057	5.250	3.365	24.486	27.159	25.211	19.094
0.110	20.418	20.418	21.990	13.444	8.012	7.211	6.249	4.006	13.156	15.957	13.490	8.188

In Step 5, polynomial equations characterizing the head-capacity characteristic curves of virtual pumps B^m_n at the node are derived analytically. At this stage of the procedure, the equations for the virtual pumps corresponding to the original physical pumps are presented, alongside those obtained by virtualizing the formulas for parallel pump combinations. The representative polynomials describing the parallel combination of virtual pumps are provided in Step 6 (Equations 14–17).

$$\text{pump1: } H_1^1 = 86.789 - 157.381Q - 4{,}654.878Q^2 \quad (14)$$

$$\text{pump2: } H_2^1 = 88.784 - 156.289Q - 4{,}598.153Q^2 \quad (15)$$

$$\text{pump3: } H_3^1 = 86.093 - 95.161Q - 5{,}135.311Q^2 \quad (16)$$

$$\text{pump4: } H_4^1 = 72.535 - 27.737Q - 5{,}065.937Q^2 \quad (17)$$

In this final step, the virtual pumps are combined in parallel through the sequence of secondary nodes toward the main node. This operation will be performed analytically, as in the previous step. Table 3 presents the data for the parallel coupling process of the virtual pumps in subsystems 1 and 2, respectively. Only the portion of the curve representing the effective combination of the two pumps in parallel is shown in this table.

Table 3. Combination of virtual rotodynamic pumps in parallel for subsystems 1 and 2

$H^1_{1\text{-}2}$ (m)	Q_{v1} (m^3/s)	Q_{v2} (m^3/s)	Q_{vP} (m^3/s)	$H^1_{3\text{-}4}$ (m)	Q_{v3} (m^3/s)	Q_{v4} (m^3/s)	Q_{vP} (m^3/s)
83.846	0.013	0.020	0.033	68.503	0.026	0.050	0.076
79.985	0.025	0.030	0.055	61.892	0.043	0.060	0.103
75.195	0.036	0.040	0.076	54.255	0.057	0.070	0.127
69.482	0.046	0.050	0.096	45.595	0.070	0.080	0.150
62.848	0.057	0.060	0.117	35.913	0.082	0.090	0.172
55.297	0.067	0.070	0.137	25.211	0.094	0.100	0.194
46.831	0.077	0.080	0.157	13.490	0.105	0.110	0.215
37.451	0.087	0.090	0.177	0.751	0.116	0.120	0.236
27.159	0.098	0.100	0.198	-	-	-	-
15.957	0.108	0.110	0.218				
3.846	0.118	0.120	0.238				

The representative polynomials for the parallel combination of the rotodynamic pumps of the two subsystems are presented below:

Subsystem 1 (Pumps 1 and 2 – NS_1 – NP): $H^1_{1\text{-}2} = 87.767 - 78.396Q - 1{,}156.246Q^2$ (18)

Subsystem 2 (Pumps 3 and 4 – NS_2 – NP): $H^1_{3\text{-}4} = 76.747 - 7.616Q - 1{,}329.363Q^2$ (19)

From this point, a virtual reduction of the overall hydraulic system begins. The resulting simplified system consists of: secondary nodes 1 and 2 – main node – discharge tank (subsystem 3). The analogy arises from the assumption that the secondary nodes contain pumping stations whose virtual pumps have characteristic head-capacity curves represented by equations 18 and 19. These pumps supply water to the main node, which then supplies water to the discharge tank. In this way, the subsystems upstream of both secondary nodes are "reduced", resulting in a "virtual simplification" of the overall hydraulic system.

Table 4 presents the results of the virtualization process for the virtual pumps located at the secondary nodes, $B^1_{1\text{-}2}$ and $B^1_{3\text{-}4}$ (virtual pumps resulting from the parallel combination of virtual pumps 1 and 2, and 3 and 4, respectively). Table 5 shows the values obtained from the parallel combination of virtual pumps $B^1_{1\text{-}2}$ and $B^1_{3\text{-}4}$, as shown in the previous table.

These values correspond to the portion of the curve representing the practical combination of the two virtual pumps in parallel.

Table 4. Data of the characteristic curves of the virtual pumps B^1_{1-2} and B^1_{3-4} and their virtualization process up to the main node

Flow rate	Flow rate	Head of virtual pumps (at the secondary nodes)		Head losses in the independent piping system of each pump		Dynamic water level elevation in the (virtual) well		Elevation of the virtual water level	Head of virtual pumps (at the main node)	
Q_{vP} (m³/s)	Q_{vP} (m³/s)	H^1_{1-2} (m)	H^1_{3-4} (m)	h_{fCS1} (m)	h_{fCS2} (m)	Z_{vNS1} (m)	Z_{vNS2} (m)	Z_{sv} (m)	H^2_{1-2} (m)	H^2_{3-4} (m)
0.033	0.076	83.846	68.503	0.876	1.446	9.250	9.250	9.250	82.970	67.057
0.055	0.103	79.985	61.892	2.147	2.543	9.250	9.250	9.250	77.838	59.348
0.076	0.127	75.195	54.255	3.850	3.734	9.250	9.250	9.250	71.345	50.521
0.096	0.150	69.482	45.595	5.959	5.045	9.250	9.250	9.250	63.522	40.550
0.117	0.172	62.848	35.913	8.456	6.483	9.250	9.250	9.250	54.392	29.431
0.137	0.194	55.297	25.211	11.330	8.049	9.250	9.250	9.250	43.967	17.163
0.157	0.215	46.831	13.490	14.572	9.743	9.250	9.250	9.250	32.259	3.747
0.177	0.236	37.451	0.751	18.173	11.565	9.250	9.250	9.250	19.278	-10.814
0.198		27.159		22.129		9.250	9.250	9.250	5.031	
0.218		15.957		26.434		9.250	9.250	9.250	-10.476	
0.238		3.846		31.083		9.250	9.250	9.250	-27.238	

Table 5. Parallel combination of virtual rotodynamic pumps B^1_{1-2} and B^1_{3-4}

H^2_{1-4} (m)	Q_{v1-2} (m³/s)	Q_{v3-4} (m³/s)	Q_{v1-4} (m³/s)
67.057	0.087	0.076	0.163
63.522	0.096	0.089	0.186
54.392	0.117	0.117	0.234
43.967	0.137	0.143	0.280
32.259	0.157	0.167	0.324
19.278	0.177	0.190	0.368
5.031	0.198	0.213	0.411

As a result, the characteristic polynomial of the parallel combination of these virtual pumps is obtained:

Subsystem 3 (NS_1 and NS_2 – NP – T): $H^2_{1-4} = 81.099 - 20.558Q - 400.682Q^2$ (20)

In this final stage of the process, the overall hydraulic system has been simplified down to the main node. The final equivalent hydraulic system would be that of a virtual pumping station whose pump would have a head-capacity curve represented by the polynomial in equation (20), pumping against the discharge tank. Next, the flow rate through the main pipeline is calculated using the equations of the resulting system (main node − pipeline − discharge tank) (21) and the equation representing the resulting curve in parallel (20).

System (NP – T): $H_{sist} = 40.750 + K_{W-D}Q^2$ (21)

Another way to determine this flow rate would be to continue applying the virtual pump method, using a further reduction process on the main node – pipeline – discharge tank system. In this case, the head losses in the main pipeline would first be subtracted from equation (20), followed by the static correction, which involves subtracting the difference between the piezometric heads at the discharge tank and the virtual suction head established for the system. For either method, the total system flow rate is: Q_{CM} = 0.264241 m³/s = 264.241 L/s and the operating head is 47.711 m.

Next, the flow rates contributed by each pump to the system are determined, as well as the respective flow velocities for each pipe. During this process, the piezometric heads of the nodes in the system are calculated. This process can be carried out graphically, as proposed by Galguera (2015) and Martínez and Riaño, or analytically, which is described in this example.

This calculation sequence will be performed from downstream to upstream, that is, by reversing the methodology. Consequently, the first calculation will be the piezometric head of the main node of the overall hydraulic system. For this, Bernoulli's equation will be applied from the main node to the discharge reservoir, according to expression (22):

$$CP_{NP} = Z_t + hf_{CP} \qquad (22)$$

Where: CP_{NP}: piezometric head at the main node of the general hydraulic system, (m); piezometric head at the outlet of the discharge pipe in the regulating tank, (m), and hf_{CP}: head losses in the main conveyance pipeline (m).

Working with equation (22), we first obtain head losses for a flow rate of 264.241 L/s of 6.961 m. The piezometric head of the main node will be 56.961 m. Next, we proceed to calculate the flow rates of each of the secondary pipes that feed into the main node. For this, we will use equations (23) and (24), which describe the virtual head-capacity curves resulting from the virtualization process of equations (18) and (19), respectively. Substituting the operating head value of $H_{1\text{-}2}{}^2 = H_{3\text{-}4}{}^2 = 47.711$ m in both equations, the respective flow rates for each subsystem are obtained: $Q_{CS1} = 0.130071$ m^3/s = 130.071 L/s and $Q_{CS2} = 0.134111$ m^3/s = 134.111 L/s, the sum of these being: $Q_{CP} = 0.264182$ m^3/s = 264.182 L/s.

$$\text{Subsystem 1 (Pumps 1 and 2 – NS}_1\text{ – NP): } H^2_{1\text{-}2} = 88.194 - 99.971Q - 1{,}624.251Q^2 \qquad (23)$$

$$\text{Subsystem 2 (Pumps 3 and 4 – NS}_2\text{ – NP): } H^2_{3\text{-}4} = 77.020 - 17.333Q - 1{,}500.290Q^2 \qquad (24)$$

With these flow rates, the piezometric heads of secondary nodes 1 and 2 are calculated. Applying an analogous equation to expression (22), the aforementioned piezometric heads are obtained: $CP_{NS1} = 67.257$ m and $CP_{NS2} = 61.061$ m

$$CP_{NSn} = CP_{NP} + hf_{CSn} \qquad (25)$$

Where: CP_{NSn}: piezometric head at secondary node n of the general hydraulic system, (m), and hf_{CSn}: head losses in secondary conduit n, (m).

Finally, the flow rates supplied by each pump to its respective secondary nodes are calculated. In this case, when the nodes to which the pumps directly supply water are reached, the individual flow rates of these pumps can be determined using the same procedure employed to determine the flow rates in the secondary lines, or directly by applying Bernoulli's equation between the suction wells (pumping stations) and their respective receiving nodes. Using this latter method, the individual flow rates of each pump are determined using equation (26). Table 6 presents the values for the general solution of the hydraulic system.

$$Z_{sn} + H_{Bn} = Z_{sn} + A_n + B_nQ + C_nQ^2 = CP_{NSn} + hf_{TISn} \qquad (26)$$

Table 6. Hydraulic information of the general solution of the hydraulic system

Pipe Section	Q (L/s)	V (m/s)	N_R (dimensionless)	f (dimensionless)	Piezometric Head (m)
TIS_1	63.511	1.049	291,299.51	0.014558	CP_{NS1} (m) 67.257
TIS_2	66.548	1.099	305,229.76	0.014434	
TIS_3	72.944	1.205	334,566.47	0.014198	CP_{NS2} (m) 61.061
TIS_4	61.274	1.012	281,041.01	0.014653	
CS_1	130.071	1.333	469,951.76	0.013347	CP_{NP} (m) 56.961
CS_2	134.111	1.086	430,549.16	0.013541	
CM and CP	264.241	1.091	605,984.51	0.012734	$CP_t = Z_t = 50.00$ m

3 Conclusion

The virtual pump method is a practical and easily implemented procedure for performing hydraulic simulations of pumped water supply systems, particularly those comprised of well fields. A virtual pump is defined as an imaginary machine consisting of the actual pump supplied by the manufacturer and all the fittings and pipes up to the point of connection (node) with the discharge pipe. This concept of virtual pumps greatly simplifies the hydraulic calculations at the actual operating points and, in general, the physical understanding of the problem. This paper provides an introduction to the method and its fundamental hydraulic principles.

The virtualization procedure can be summarized as follows: using the pump head-capacity characteristic curves and the piping system curves from the pumps to the piping system junction, the latter are subtracted from the former, thus obtaining the pump characteristic curves adjusted to the head corresponding to the piping system junction. These "reduced" curves up to the supply node are the virtual head-flow characteristic curves of the virtual pumps. After this first step, the conventional method of summing the adjusted curves can then be applied, since they are all referenced to the same head.

Then, by analyzing the system in reverse (from the surge tank to the pumping stations), the flow rate delivered by each pump at the pumping stations is obtained. The same practical example presented in Part 1 of this article is used to illustrate this methodology more effectively, demonstrating its advantages and practical utility in the hydraulic simulation of pumped water supply systems.

Conflicts of interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Cabrera E. (2009). "Ingeniería hidráulica aplicada a los sistemas de distribución de agua", Editorial Unidad Docente Mecánica de Fluidos, Universidad Politécnica de Valencia, t. 1 y 2, 3ra edición, ISBN 978-846-13-3949-5, Valencia, España.

[2] Castilla A. y Galvis G (1993). "Bombas y estaciones de bombeo", Centro Inter-Regional de Abastecimiento y Remoción de Agua (CINARA)-Universidad del Valle, Ed. Ultragraf Editores, ISBN 978-777-98-6726-5, Cali, Colombia.

[3] Fuertes V. S et al. (2002). "Modelación y diseño de redes de abastecimiento de agua", Editorial Grupo de Mecánica de los Fluidos, Universidad Politécnica de Valencia, 1ra edición, ISBN 84-89487-06-5, Valencia, España.

[4] Galguera C. L. (2015). "Metodología para el diseño y simulación hidráulica de un sistema fuente por bombeo desde campo de pozos", Trabajo de diploma, Facultad de Ingeniería Civil, Instituto Superior Politécnico "José Antonio Echeverría", Cujae, La Habana, Cuba.

[5] Instituto Nacional de Recursos Hidráulicos (INRH) (2006). "Instructivo para la utilización de Tuberías y Accesorios de PEAD", 63 pp., La Habana, Cuba.

[6] Martínez Y. (2011). "Metodología para el diseño hidráulico de las estaciones de bombeo para acueducto", Tesis de doctorado, Instituto Superior Politécnico "José Antonio Echeverría" (Cujae), La Habana, Cuba.

[7] Martínez Y. y Riaño F. (2010). "Características peculiares de la operación de bombas rotodinámicas en paralelo", Ciencias Técnicas Agropecuarias, 19 (2): 38-43, Universidad Agraria de La Habana "Fructuoso Rodríguez" (UNAH), ISSN 1010-2760, Mayabeque, Cuba.

[8] Martínez Y. y Riaño F. (2024). "Método de las bombas rotodinámicas virtuales", Ingeniería Hidráulica, XLV (4): 55-67, Universidad Tecnológica de La Habana "José Antonio Echeverría", Cujae, ISSN 2788-6050, La Habana, Cuba.

[9] Martínez Y. y Riaño F. (2025). "Simulación hidráulica de sistemas fuentes por bombeo por campos de pozos: 1ra Parte", Ingeniería Hidráulica, XLVI (1): 45-59, Universidad Tecnológica de La Habana "José Antonio Echeverría", Cujae,

ISSN 2788-6050, La Habana, Cuba.

[10] Martínez Y. and Riaño F. (2025). "Virtual rotodynamic pumps method", Journal Region-Water Conservancy, 8 (2): 151-161, ISSN 2630-4902, Singapur, Singapur.

[11] Pardo R. y Ruiz M. I. (1980). "Algunas consideraciones sobre el funcionamiento de bombas en paralelo", *Revista Ingeniería Hidráulica*, 1 (1): 20-28, Instituto Superior Politécnico Superior "José Antonio Echeverría", La Habana, Cuba.

[12] Sterling SIHI (2003). "Principios Básicos para el Diseño de Instalaciones de Bombas Centrífugas", (Manual técnico), 7th edition, Ed. Sterling Fluid Systems Group, Madrid, España, 2003.

[13] Turiño, I. M. (1996). "Procedimientos metodológicos para el diagnóstico operacional en sistemas de bombeo mediante modelos matemáticos", Tesis de doctorado, Facultad de Ingeniería Mecánica, Universidad Central de las Villas Marta Abreu (UCLV), Santa Clara, Cuba.

Authors' Contribution

Yaset Martínez Valdés https://orcid.org/0000-0001-9770-022X

She participated in developing the methodological procedure for applying the Virtual Pump Method, as well as in processing the case study data, contributing to its analysis and interpretation. She also participated in the search for bibliographic references and in writing the paper. email: yaset1977@gmail.com

Félix Abelis Riaño Valle https://orcid.org/0000-0002-9342-6064

She made contributions to the research design and the analysis of the results of the case study, as well as to the search for bibliographic information, review and writing of the work in its final version. email: riano@tesla.cujae.edu.cu